Part 3
Chapter 8

Linear Algebraic Equations and Matrices

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Part Overview

- In Part Two, we determined the value x that satisfied a single equation f(x)=0.
- Now we deal with finding $x_1, x_2, \ldots, x_n$ that simultaneously satisfy a set of equations:

$$f(x_1, x_2, \ldots, x_n) = 0$$
$$f(x_1, x_2, \ldots, x_n) = 0$$
$$\vdots$$
$$f(x_1, x_2, \ldots, x_n) = 0$$

- In Part Three, we deal with linear algebraic equations that are of the general form

$$\begin{aligned}
a_{11}x_1 &+ a_{12}x_2 &+ \cdots &+ a_{1n}x_n &= b_1 \\
a_{21}x_1 &+ a_{22}x_2 &+ \cdots &+ a_{2n}x_n &= b_2 \\
&\vdots & & \vdots & \\
a_{n1}x_1 &+ a_{n2}x_2 &+ \cdots &+ a_{nn}x_n &= b_n
\end{aligned}$$

where *a's and b's are constants*

Part Overview (cont.)

- Chap. 8: Matrix algebra
- Chap. 9: Gauss Elimination
  - Naïve gauss elimination
  - Partial pivoting
  - Tridiagonal matrices
- Chap. 10: LU Factorization
- Chap. 11: Matrix Inverse
- Chap. 12: Iterative Methods
  - Gauss Seidel and Jacobi method

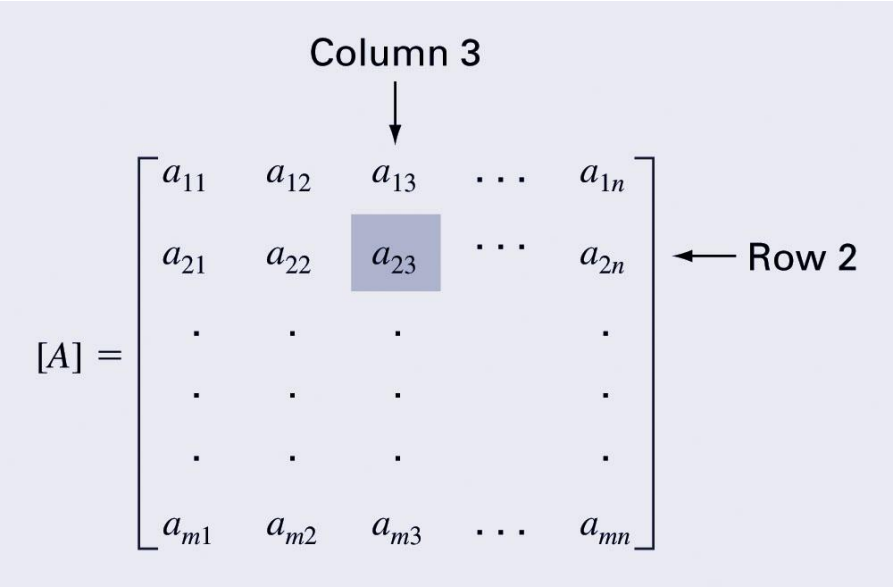
Chapter Objectives

- Understanding matrix notation.
- Being able to identify the following types of matrices:

  identify, diagonal, symmetric, triangular, and tridiagonal.
- Knowing how to perform matrix multiplication and being able to assess when it is feasible.
- Knowing how to represent a system of linear equations in matrix form.
- Knowing how to solve linear algebraic equations with left division and matrix inversion in MATLAB.

Overview

- A matrix consists of a rectangular array of elements represented by a single symbol (example: [A]).
- An individual entry of a matrix is an element (example: a_{23})



Overview (cont)

- A horizontal set of elements is called a row and a vertical set of elements is called a column.
- The first subscript of an element indicates the row while the second indicates the column.
- The size of a matrix is given as m rows by n columns, or simply m by n (or m x n).
- 1 x n matrices are row vectors.
- m x 1 matrices are column vectors.

Special Matrices

- Matrices where m=n are called square matrices.
- There are a number of special forms of square matrices:

Symmetric

$$[A]=\begin{bmatrix}5 & 1 & 2\\ 1 & 3 & 7\\ 2 & 7 & 8\end{bmatrix}$$

Diagonal

$$[A]=\begin{bmatrix}a_{11} & & \\ & a_{22} & \\ & & a_{33}\end{bmatrix}$$

Identity

$$[A]=\begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix}$$

Upper Triangular

$$[A]=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ & a_{22} & a_{23}\\ & & a_{33}\end{bmatrix}$$

Lower Triangular

$$[A]=\begin{bmatrix}a_{11} & & \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

Banded

$$[A]=\begin{bmatrix}a_{11} & a_{12} & & \\ a_{21} & a_{22} & a_{23} & \\ & a_{32} & a_{33} & a_{34}\\ & & a_{43} & a_{44}\end{bmatrix}$$

Matrix Operations

- Two matrices are considered equal if and only if every element in the first matrix is equal to every corresponding element in the second. This means the two matrices must be the same size.

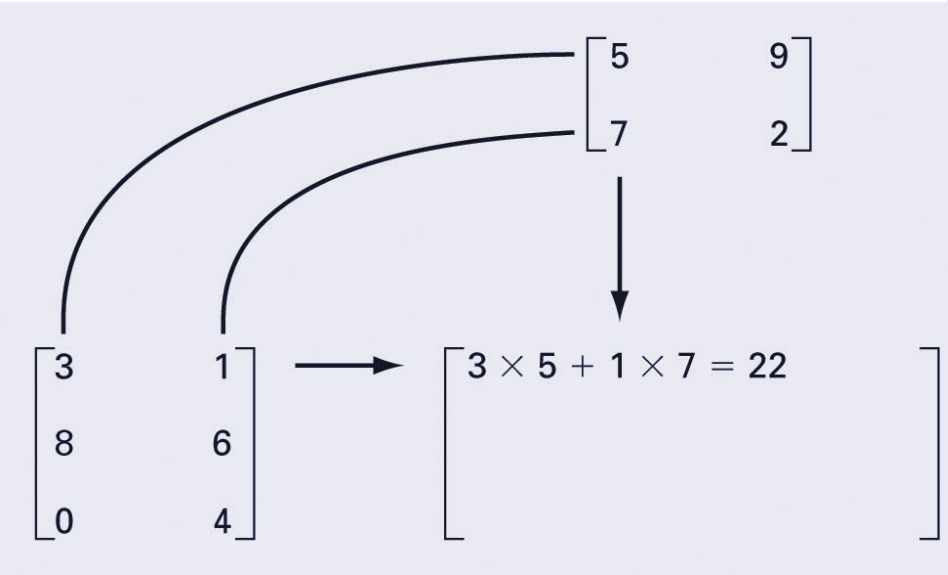
- Matrix addition and subtraction are performed by adding or subtracting the corresponding elements. This requires that the two matrices be the same size.

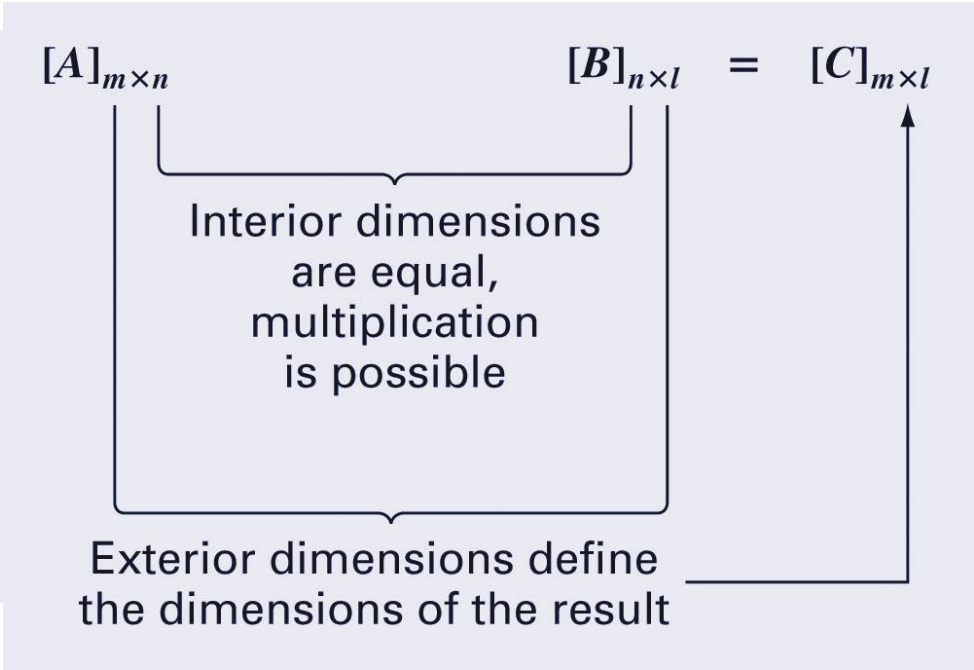
- Scalar matrix multiplication is performed by multiplying each element by the same scalar.

Matrix Multiplication

- The elements in the matrix [C] that results from multiplying matrices [A] and [B] are calculated using:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$





Matrix Inverse and Transpose

- The inverse of a square, nonsingular matrix [A] is that matrix which, when multiplied by [A], yields the identity matrix.

$$[A][A]^{-1} = [A]^{-1}[A] = [I]$$

- The transpose of a matrix involves transforming its rows into columns and its columns into rows.

$(a_{ij})^T = a_{ji}$

Representing Linear Algebra

- Matrices provide a concise notation for representing and solving simultaneous linear equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$[A]\{x\} = \{b\}$$

Solving With MATLAB

- MATLAB provides two direct ways to solve systems of linear algebraic equations [A]{x}={b}:

  - Left-division

    x = A\b

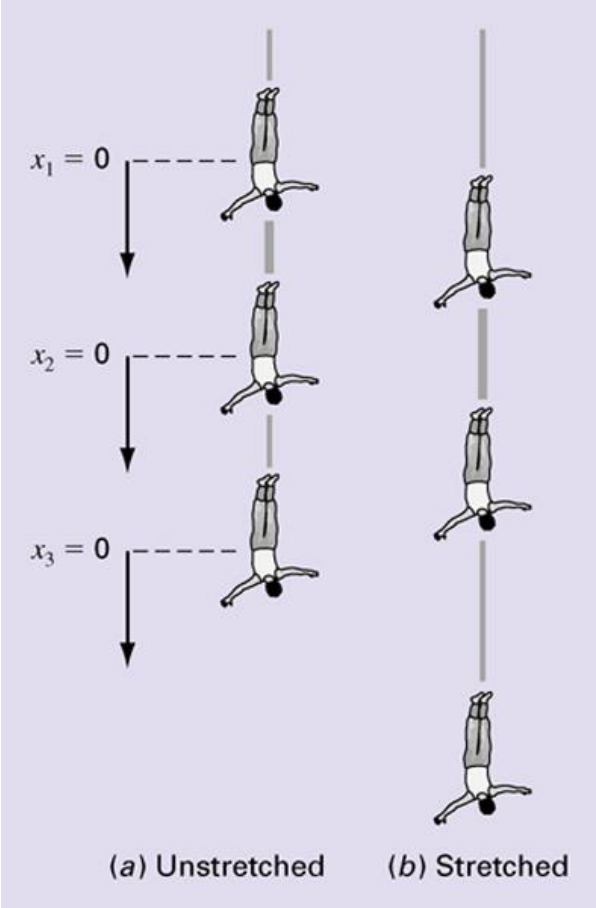
  - Matrix inversion

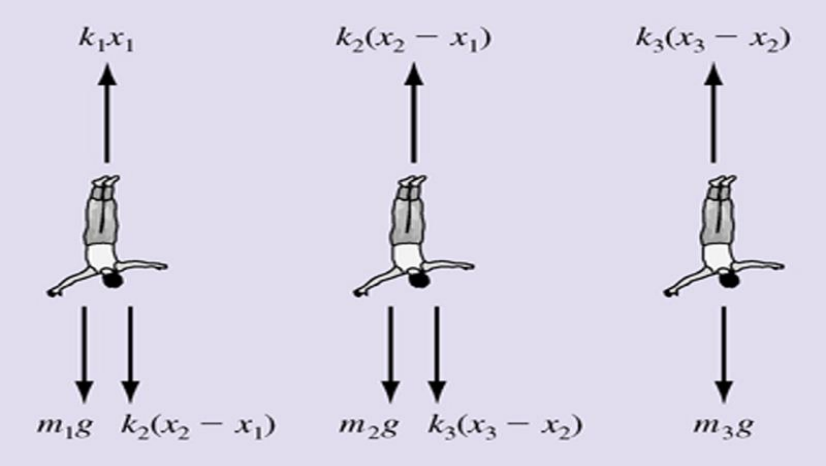
    x = inv(A)*b

- The matrix inverse is less efficient than left-division and also only works for square, non-singular systems.

Example 8.2 (1/3)

Q. Use Matlab to solve the bungee jump problem described at the beginning of this chapter. The parameters are





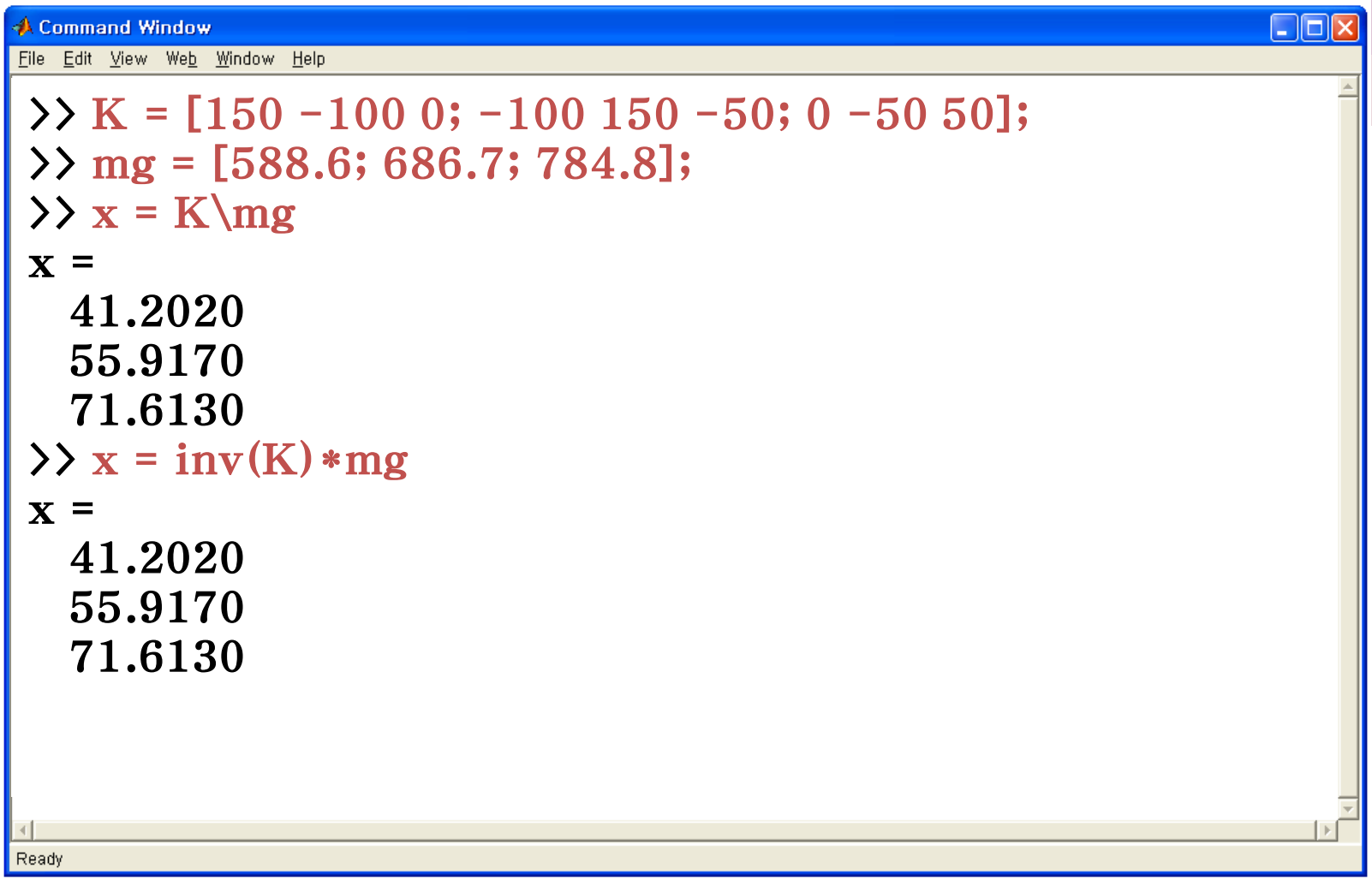
$$m_1 g + k_2(x_2 - x_1) - k_1 x_1 = 0$$

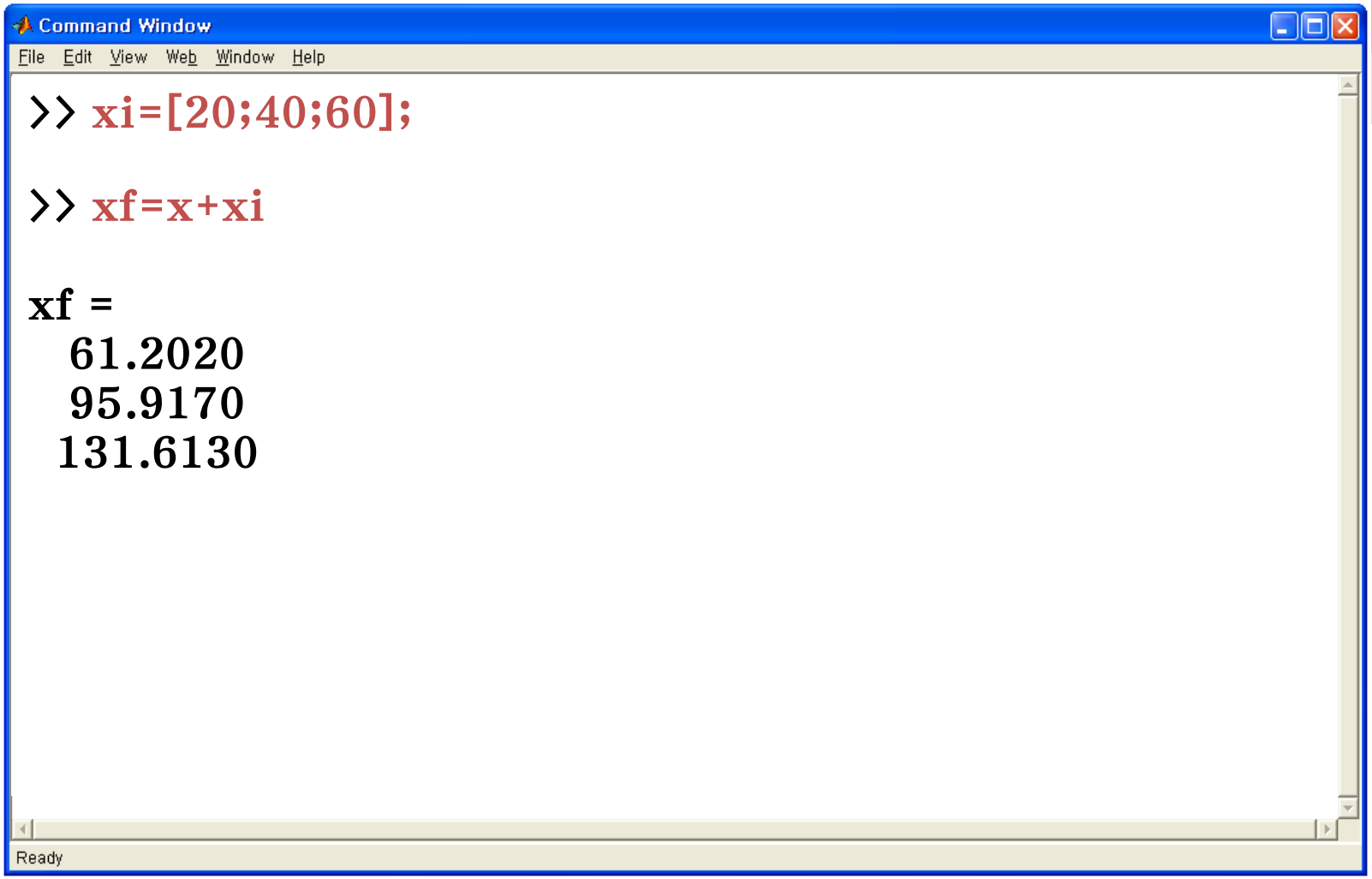
$$m_2 g + k_3(x_3 - x_2) - k_2(x_2 - x_1) = 0$$

$$m_3 g - k_3(x_3 - x_2) = 0$$

Jumper	Mass, m (kg)	Spring Constant, k (N/m)	Unstretched Cord Length, xi (m)
top(1)	60	50	20
middle(2)	70	100	20
bottom(3)	80	50	20

$$\begin{bmatrix} 150 & -100 & 0 \\ -100 & 150 & -50 \\ 0 & -50 & 50 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 588.6 \\ 686.7 \\ 784.8 \end{Bmatrix}$$


Command Window
File Edit View Web Window Help
>> K = [150 -100 0; -100 150 -50; 0 -50 50];
>> mg = [588.6; 686.7; 784.8];
>> x = K\mg
x =
41.2020
55.9170
71.6130
>> x = inv(K)*mg
x =
41.2020
55.9170
71.6130
Ready


Command Window
File Edit View Web Window Help
>> xi=[20;40;60];
>> xf=x+xi
xf =
61.2020
95.9170
131.6130
Ready