HW6 (Thermodynamics I)

Due: 2015/06/09, 1 PM

1. Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific entropy, using a) the steam tables, b) the ideal gas water Table A.8, and c) the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3 Superheated vapor $s_1 = 6.9299$ kJ/kgK

State 2: Table B.1.3 $s_2 = 9.7059$ kJ/kgK

$$s_2 - s_1 = 9.7059 - 6.9299 = \mathbf{2.776\ kJ/kgK}$$

b)

Table A.8 at 423.15 K: $s^o_{T1} = 11.13891$ kJ/kgK

Table A.8 at 1473.15 K: $s^o_{T2} = 13.86383$ kJ/kgK

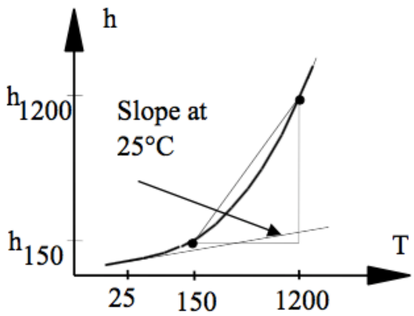
$$s_2 - s_1 = s^o_{T2} - s^o_{T1} - R \ln \frac{P_2}{P_1} = s^o_{T2} - s^o_{T1}$$

$$s^o_{T2} - s^o_{T1} = 13.86383 - 11.13891 = \mathbf{2.72492\ kJ/kgK}$$

c) Table A.5: $C_{po} = 1.872$ kJ/kgK

$$s_2 - s_1 \approx C_{po} \ln \left(\frac{T_2}{T_1}\right) = 1.872 \ln \left(\frac{1473.15}{423.15}\right) = \mathbf{2.3352\ kJ/kgK}$$

Notice how the average slope from 150°C to 1200°C is higher than the one at 25°C (= C_{po})



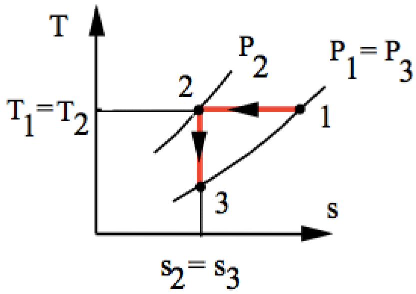
2. We wish to obtain a supply of cold helium gas by applying the following technique. Helium contained in a cylinder at ambient conditions, 100 kPa, 20°C, is compressed in a reversible isothermal process to 600 kPa, after which the gas is expanded back to 100 kPa in a reversible adiabatic process.

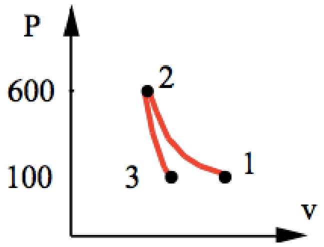
a. Show the process on a T–s diagram.

b. Calculate the final temperature and the net work per kilogram of helium.

Solution:

a)





b) The adiabatic reversible expansion gives constant s from the entropy equation Eq.6.37. With ideal gas and constant specific heat this gives relation in Eq.6.23

$$T_3 = T_2(P_3/P_2)^{\frac{k-1}{k}} = 293.15\ (100/600)^{0.4} = \mathbf{143.15\ K}$$

The net work is summed up over the two processes. The isothermal process has work as Eq.6.31

$$_1w_2 = -RT_1 \ln(P_2/P_1) = -2.0771 \text{ kJ/kg-K} \times 293.15 \text{ K} \times \ln(600/100)$$

$$= -1091.0 \text{ kJ/kg}$$

The adiabatic process has a work term from energy equation with no q

$$_2w_3 = C_{Vo}(T_2\text{-}T_3) = 3.116 \text{ kJ/kg-K } (293.15 - 143.15) \text{ K} = +467.4 \text{ kJ/kg}$$

The net work is the sum

$$w_{NET} = -1091.0 + 467.4 = \mathbf{-623.6\ kJ/kg}$$

3. Water in a piston/cylinder is at 101 kPa, 25°C, and mass 0.5 kg. The piston rests on some stops, and the pressure should be 1000 kPa to float the piston. We now heat the water from a 200°C reservoir, so the volume becomes 5 times the initial volume. Find the total heat transfer and the entropy generation.

Solution:

Take CV as the water out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$;

Energy Eq.: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.6.37: $m(s_2 - s_1) = \int dQ/T + {}_1S_{2\ gen} = {}_1Q_2/T_{res} + {}_1S_{2\ gen}$

Process: v = constant, then $P = C = P_{float}$.

Volume does go up so we get $v_2 = 5\ v_1$

State 1: $v_1 = 0.001003\ m^3/kg$, $u_1 = 104.86$ kJ/kg, $s_1 = 0.3673$ kJ/kgK

State 2: $P_2 = P_{float}$, $v_2 = 5 \times 0.001003 = 0.005015\ m^3/kg$; $T_2 = 179.91°C$

$x_2 = (v_2 - v_f)/v_{fg} = (0.005015 - 0.001127)/0.19332 = 0.02011$,

$u_2 = 761.67 + x_2 \times 1821.97 = 798.31$ kJ/kg

$s_2 = 2.1386 + x_2 \times 4.4478 = 2.2280$ kJ/kgK

From the process equation (see P-V diagram) we get the work as

$${}_1w_2 = P_{float}(v_2 - v_1) = 1000\ \text{kPa}\ (0.005015 - 0.001003)\ m^3/kg = 4.012\ \text{kJ/kg}$$

From the energy equation we solve for the heat transfer

$${}_1Q_2 = m[u_2 - u_1 + {}_1w_2] = 0.5\times[798.31 - 104.86 + 4.012] = \mathbf{348.7\ kJ}$$

$${}_1S_{2\ gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 0.5(2.2280 - 0.3673) - 348.7/473.15$$

$$= \mathbf{0.1934\ kJ/K}$$

4.

5. A room at 22°C is heated electrically with 1500 W to keep steady temperature. The outside ambient is at 5°C. Find the flux of S (= $\dot{Q}$/T) into the room air, into the ambient and the rate of entropy generation.

CV. The room and walls out to the ambient T, we assume steady state

Energy Eq.: $0 = \dot{W}_{el\ in} - \dot{Q}_{out}$ => $\dot{Q}_{out} = \dot{W}_{el\ in} = 1500$ W

Entropy Eq.: $0 = -\dot{Q}_{out}/T + \dot{S}_{gen\ tot}$

Flux of S into room air at 22°C: $\dot{Q}/T = 1500 / 295.15 = 5.08$ W/K

Flux of S into ambient air at 5°C: $\dot{Q}/T = 1500 / 278.15 = 5.393$ W/K

Entropy generation: $\dot{S}_{gen\ tot} = \dot{Q}_{out}/T = 1500 / 278.15 =$ **5.393 W/K**

Comment: The flux of S into the outside air is what leaves the control volume and since the control volume did not receive any S it was all made in the process. Notice most of the generation is done in the heater, the room heat loss process generates very little S (5.393 – 5.08 = 0.313 W/K)

6. A turbo charger boosts the inlet air pressure to an automobile engine. It consists of an exhaust gas driven turbine directly connected to an air compressor, as shown in Fig. P7.36. For a certain engine load the conditions are given in the figure. Assume that both the turbine and the compressor are reversible and adiabatic having also the same mass flow rate. Calculate the turbine exit temperature and power output. Find also the compressor exit pressure and temperature.

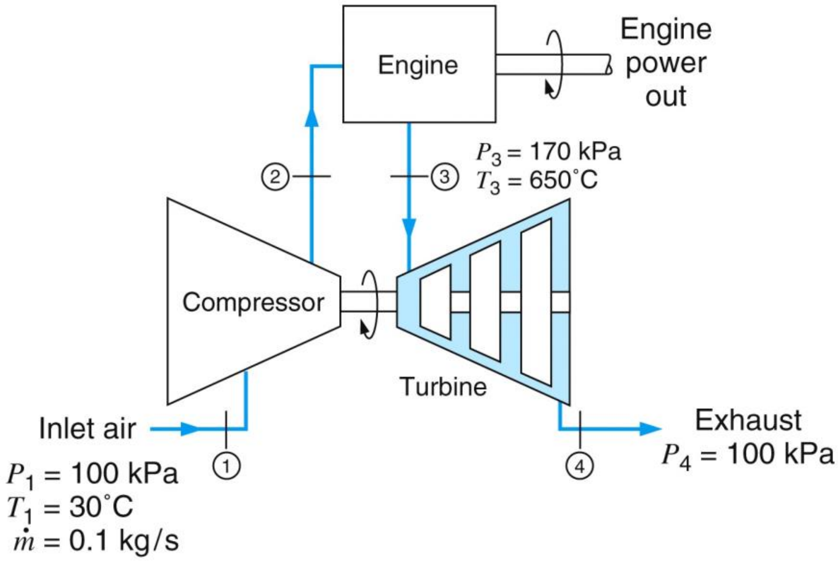


Figure P7.41

Solution:

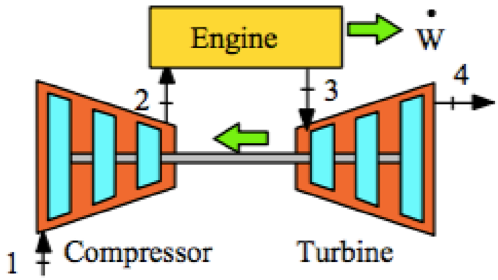
CV: Turbine, Steady single inlet and exit flows,

Process: adiabatic: $q = 0$,

reversible: $s_{gen} = 0$

EnergyEq.4.13: $w_T = h_3 - h_4$,

Entropy Eq.7.8: $s_4 = s_3$



The property relation for ideal gas gives Eq.6.23, k from Table A.5

$$s_4 = s_3 \rightarrow T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 923.2 \text{ K} \left(\frac{100}{170}\right)^{0.286} = \mathbf{793.2 \text{ K}}$$

The energy equation is evaluated with specific heat from Table A.5

$$w_T = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004(923.2 - 793.2) = 130.5 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = \mathbf{13.05 \text{ kW}}$$

C.V. Compressor, steady 1 inlet and 1 exit, same flow rate as turbine.

Energy Eq.4.13: $-w_C = h_2 - h_1$,

Entropy Eq.7.9: $s_2 = s_1$

Express the energy equation for the shaft and compressor having the turbine power as input with the same mass flow rate so we get

$$-w_C = w_T = 130.5 \text{ kJ/kg} = C_{P0}(T_2 - T_1) = 1.004(T_2 - 303.2)$$

$$T_2 = \mathbf{433.2 \text{ K}}$$

The property relation for $s_2 = s_1$ is Eq.6.23 and inverted as

$$P_2 = P_1(T_2/T_1)^{\frac{k}{k-1}} = 100 \text{ kPa} \left(\frac{433.2}{303.2}\right)^{3.5} = \mathbf{348.7 \text{ kPa}}$$

7. In a heat-driven refrigerator with ammonia as the working fluid, a turbine with inlet conditions of 2.0 MPa, 70°C is used to drive a compressor with inlet saturated vapor at –20°C. The exhausts, both at 1.2 MPa, are then mixed together. The ratio of the mass flow rate to the turbine to the total exit flow was measured to be 0.62. Can this be true?

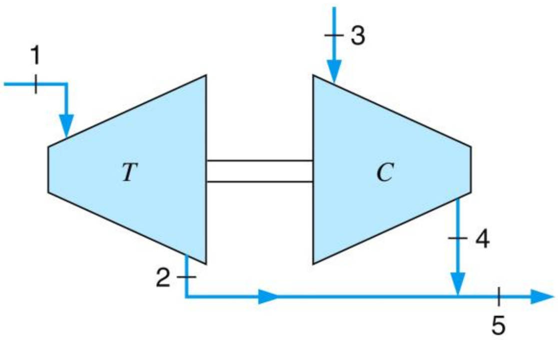
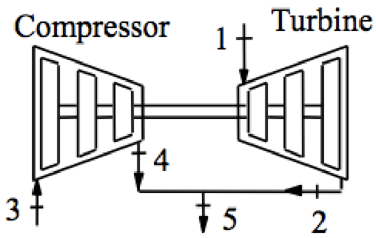


Figure P7.107

Solution:

Assume the compressor and the turbine are both adiabatic.

C.V. Total:



Continuity Eq.4.11: $\dot{m}_5 = \dot{m}_1 + \dot{m}_3$

Energy Eq.4.10: $\dot{m}_5 h_5 = \dot{m}_1 h_1 + \dot{m}_3 h_3$

Entropy: $\dot{m}_5 s_5 = \dot{m}_1 s_1 + \dot{m}_3 s_3 + \dot{S}_{C.V.,gen}$

$$s_5 = y s_1 + (1\text{-}y)s_3 + \dot{S}_{C.V.,gen}/\dot{m}_5$$

Assume $y = \dot{m}_1/\dot{m}_5 = 0.62$

State 1: Table B.2.2 $h_1 = 1542.7$ kJ/kg, $s_1 = 4.982$ kJ/kg K,

State 3: Table B.2.1 $h_3 = 1418.1$ kJ/kg, $s_3 = 5.616$ kJ/kg K

Solve for exit state 5 in the energy equation

$$h_5 = yh_1 + (1\text{-}y)h_3 = 0.62 \times 1542.7 + (1 - 0.62)1418.1 = 1495.4 \text{ kJ/kg}$$

State 5: $h_5 = 1495.4$ kJ/kg, $P_5 = 1200$ kPa $\Rightarrow$ $s_5 = 5.056$ kJ/kg K

Now check the 2nd law, entropy generation

$$\Rightarrow \dot{S}_{C.V.,gen}/\dot{m}_5 = s_5 - ys_1 - (1\text{-}y)s_3 = \mathbf{-0.1669\ Impossible}$$

The problem could also have been solved assuming a reversible process and then find the needed flow rate ratio y. Then y would have been found larger than 0.62 so the stated process can not be true.

8. An initially empty 0.1 m3 cannister is filled with R-410A from a line flowing saturated liquid at -5°C. This is done quickly such that the process is adiabatic. Find the final mass, liquid and vapor volumes, if any, in the cannister. Is the process reversible?

Solution:

C.V. Cannister filling process where: ${}_1Q_2 = \emptyset$; ${}_1W_2 = \emptyset$; $m_1 = \emptyset$

Continuity Eq.4.15: $m_2 - \emptyset = m_{in}$;

Energy Eq.4.16: $m_2u_2 - \emptyset = m_{in}h_{line} + \emptyset + \emptyset \quad \Rightarrow \quad u_2 = h_{line}$

1: Table B.4.1 $\quad u_f = 49.65,\ u_{fg} = 201.75,\ \ h_f = 50.22$ all kJ/kg

2: $P_2 = P_{line}$; $u_2 = h_{line} \Rightarrow$ 2 phase $u_2 > u_f$; $u_2 = u_f + x_2u_{fg}$

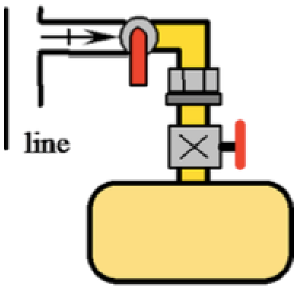
$x_2 = (50.22 - 49.65)/201.75 = 0.002825$

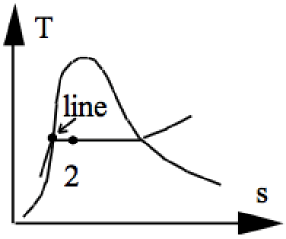
$\Rightarrow v_2 = v_f + x_2v_{fg} = 0.000841 + 0.002825 \times 0.03764 = 0.0009473\ m^3/kg$

$\Rightarrow m_2 = V/v_2 = \mathbf{105.56\ kg}$; $m_f = 106.262$ kg; $m_g = 0.298$ kg

$V_f = m_f v_f = \mathbf{0.089}\ m^3$; $V_g = m_g v_g = \mathbf{0.0115\ m^3}$

Process is irreversible (throttling) $s_2 > s_f$





9. Steam enters a turbine at 200°C, 0.5 MPa and the mass flow rate is 2 kg/s. The steam leaves the turbine at 0.1 MPa. For simple analysis, ignore the kinetic energy at the inlet and the outlet. Calculate the power of the turbine (a) when the process is isentropic and (b) when the efficiency of the turbine is 85%.

시할 수 있는 것으로 ... $\dot{W}_t = \dot{m}w_t$, $s_{es} = s_i$ ... 있다.

$$h_i = h_e + w_t,$$

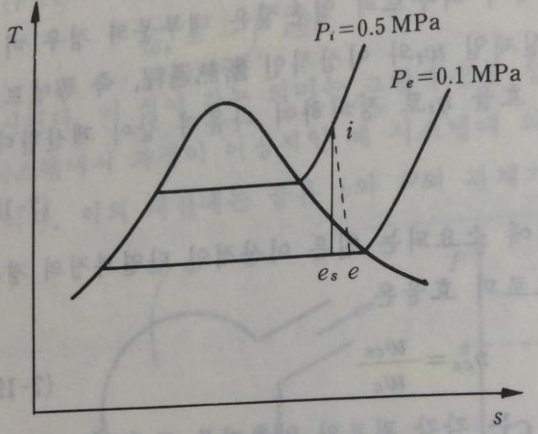


그림 E 7-1

(1) $P_i = 0.5$ MPa, $T_i = 200$℃에서

$$h_i = 2855.4 \text{ kJ/kg}, \quad s_i = 7.0592 \text{ kJ/kg·K}$$

$P_e = 0.1$ MPa, $s_{es} = s_i = 7.0592$ kJ/kg·K에서 s_{es}가 $P_e = 0.1$ MPa에서의 $s_{fe} =$ 1.3026 kJ/kg·K와 $s_{ge} = 7.3594$ kJ/kg·K 사이에 있으므로 터어빈의 이상적인 출구 상태 e_s는 濕蒸氣상태이다. 이 때의 건도는 다음과 같이 계산된다.

$s_{es} = s_{fe} + x_{es}s_{fge}$에서

$$7.0592 = 1.3026 + x_{es}(6.0568), \quad x_{es} = 0.9504$$

$$h_{es} = h_{fe} + x_{es}h_{fge} = 417.46 + (0.9504)(2258.0)$$
$$= 2563.46 \text{ kJ/kg}$$

$$w_{ts} = h_i - h_{es} = 2855.4 - 2563.46 = 291.94 \text{ kJ/kg}$$

$$\dot{W}_{ts} = \dot{m}w_{ts} = (2)(291.94) = 583.88 \text{ kW}$$

(2) 터어빈 효율의 정의

$$\eta_t = \frac{w_t}{w_{ts}} = \frac{h_i - h_e}{h_i - h_{es}}$$

에서

$$w_t = h_i - h_e = \eta_t w_{ts} = 248.15 \text{ kJ/kg}$$

$$\dot{W}_t = \dot{m}w_t = 496.3 \text{ kW}$$

한편 터어빈의 출구상태는 위의 식에서

$$h_e = h_i - \eta_t(h_i - h_{es}) = 2607.25 \text{ kJ/kg}$$

이므로, $P_e = 0.1$ MPa, $h_e = 2607.25$ kJ/kg으로부터

$$h_e = h_{fe} + x_e h_{fge} = 417.46 + x_e(2258.0)$$

$$x_e = 0.9698$$

따라서 출구의 엔트로피는

$$s_e = s_{fe} + x_e s_{fge}$$
$$= 1.3026 + (0.9698)(6.0568) = 7.1765 \text{ kJ/kg·K}$$

로서 실제의 단열과정에서 출구의 엔트로피가 증가되었음을 확인할 수 있다.

10. Air of 80°C and 0.5 MPa flows at 2 kg/s in a pipe. As the air flow through a valve, the pressures decreases to 0.2 MPa. Calculate the temperature, speed, and entropy generation rate of the air at the exit of the valve. Assume that the pipe wall is well insulated and the specific heat of the air is constant.

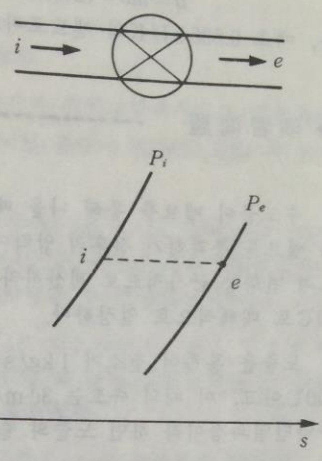


그림 E 7-4

풀이 밸브 전후의 상태를 i와 e로 그림 E 7-4와 같이 표시하자.

$$h_i+\frac{1}{2}V_i^2 = h_e+\frac{1}{2}V_e^2$$

$$v_i = \frac{RT_i}{P_i} = \frac{(0.287)(353)}{500} = 0.2026\ \mathrm{m^3/kg}$$

$$V_i = \frac{\dot{m}v_i}{A} = \frac{(2)(0.2026)}{(\pi/4)(0.2)^2} = 12.899\ \mathrm{m/s}$$

$v_e = \dfrac{RT_e}{P_e}$ 이므로

$$V_e = \frac{\dot{m}v_e}{A} = \frac{(2)(0.287)/200\times T_e}{(\pi/4)(0.2)^2} = 0.0913\,T_e\ \mathrm{m/s}$$

$h_i+\frac{1}{2}V_i^2 = h_e+\frac{1}{2}V_e^2$ 또는 $C_pT_i+\frac{1}{2}V_i^2 = C_pT_e+\frac{1}{2}V_e^2$에서

$$1.0035\times 353+\frac{1}{2}(12.899)^2\times 10^{-3} = 1.0035\,T_e+\frac{1}{2}(0.0913\,T_e)^2\times 10^{-3}$$

$$354.2+0.0832 = 1.0035\,T_e+4.17\times 10^{-6}\,T_e^2$$

위의 식에서 T_e를 직접 구할 수도 있으나 운동에너지항을 무시할 수 있음을 감안하면 $T_e \approx T_i = 353\ \mathrm{K}$이다. 운동에너지항을 완전히 무시한다면 $T_e = T_i$임이 분명하며, 이는 앞에서 설명한 스로틀링에서 엔탈피가 $h_i = h_e$로 변하지 않는 특수한 예에 속한다.

$$V_e = (0.0913)(353) = 32.2\ \mathrm{m/s}$$

밸브를 통과하는 과정은 비가역과정이나 엔트로피의 변화는 이상적인 과정을 선택하여 계산할 수 있다. 즉 $T_i = 353\ \mathrm{K}$, $P_i = 0.5\ \mathrm{MPa}$과 $T_e = T_i = 353\ \mathrm{K}$, $P_e = 0.2\ \mathrm{MPa}$ 사이의 엔트로피 변화를 구하면 되며, 이것이 生成엔트로피가 된다.

$$\theta = s_e - s_i = C_p \ln\frac{T_e}{T_i} - R\ln\frac{P_e}{P_i}$$

$$= 0-(0.287)\ln\left(\frac{0.2}{0.5}\right)=0.263\ \mathrm{kJ/kg\cdot K}$$

$$\dot{\theta}=\dot{m}\theta=(2)(0.263)=0.526\ \mathrm{kJ/K\cdot s}$$

즉, 매초 0.526 kJ/K의 엔트로피가 생성된다.

▶ 演習問題

7-1 수도물이 밸브를 통해 나올 때 스로틀링 과정으로 近似化할 수 있다. 수도물이 밸브를 통과하기 전후의 압력이 각각 0.5 MPa, 0.1 MPa일 때 물 1 kg당 엔트로피 변화를 근사적으로 계산하라. 물의 비체적은 0.001 $\mathrm{m^3/kg}$이고, 물의 온도는 20℃로 대체적으로 일정하다.

7-2 노즐을 통하여 질소가 1 kg/s의 유량으로 흐르고 있다. 입구상태는 400 kPa, 200℃이고, 이 때의 속도는 30 m/s이다. 출구의 압력이 100 kPa일 때 이 과정을 가역단열과정이라 하면 노즐의 출구에서 속도와 출구면적을 구하라.

7-3 증기터어빈의출력이 2 MW이다. 터어빈 입구의 수증기상태는 압력 3.0 MPa, 온도 260℃이고, 출구압력은 200 kPa이다. 터어빈효율은 0.85이다.

(a) 요구되는 증기의 유량은 얼마인가?

(b) 출구 수증기의 乾度는 얼마인가?

(c) 터어빈의 엔트로피 생성량은 얼마인가?

7-4 노즐에서 수증기가 6 MPa, 360℃로부터 150 kPa로 可逆斷熱膨脹을 한다. 만일 입구속도가 매우 작다면 수증기의 출구속도는 얼마가 되는가?

7-5 공기가 분당 8 kg의 양으로 입구속도를 무시할 정도로 노즐로 들어가 압력이 4 bar에서 2.2 bar로 팽창한다. 이 과정에서 온도는 900℃에서 750℃로 떨어졌다. 노즐을 떠나는 공기의 속도, 팽창하는 데 마찰이 없다면 공기가 도달할 수 있는 속도, 노즐효율을 구하라. 또 이 과정에서 매분당의 엔트로피 생성량을 계산하라.

7-6 헬륨이 초기압력과 온도가 10 bar, 400 K에서 최종압력 1 bar로 "$PV^{1.5}$=일정"인 可逆폴리트로픽 과정으로 팽창할 때 [illegible] 계산하라. 이 때 헬륨은 理想氣體 [illegible]