Numerical Analysis (수치해석)

Course#: 47892

Mechanical Engineering

Chung-Ang University

- Instructor
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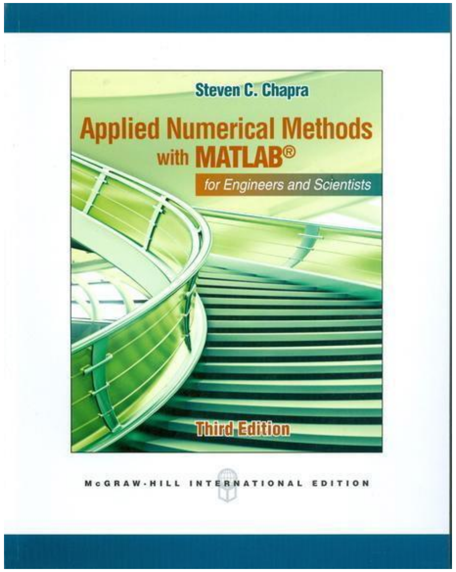
  - Office hour: WED, 14:00 – 16:00
- Prerequisites
  - Engineering Mathematics, Calculus
- Language
  - Korean
- Textbook
  - *Applied Numerical Methods with MATLAB*, Chapra

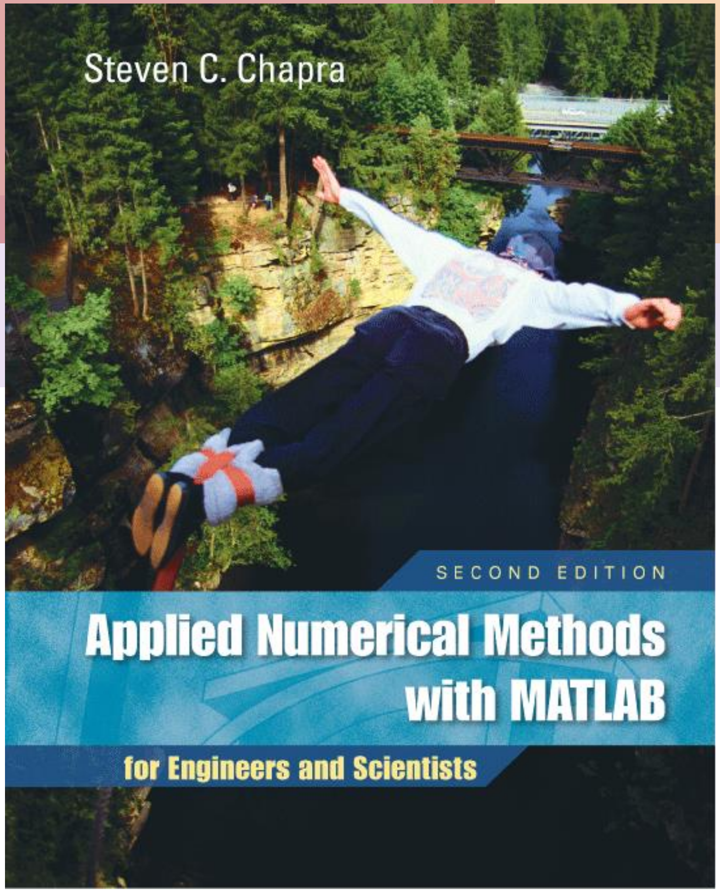


- Assessment
  - mid term(45%) / final exam(45%) / homework (5%) / Attendance (0 %)/ Attitude, participation(5%)

- Exams
  - ~~Closed book exams~~
  -
  - ~~Only the official calculator designated by the department is allowed for the use in exams.~~
  - Will necessarily give at least one problem that is *very similar* to homework problems.
  - You can be wrong with digits, but you cannot about how to solve. Marking/grading will evaluate every step of your answer.

Course schedule (tentative)

Week	Topic	Note
1	**Introduction**	
2	**Roundoff and truncation erros**	
3	**Bracketing and Open methods**	
4	**Linear algebraic equations and matrices**	
5	**Gauss elimination**	
6	**LU factorization**	
7	**Matrix inverse and condition**	
8	**Mid-term exam**	midterm exam
9	**Iterative methods**	
10	**Linear regression/Generalizaed linear least squares**	
11	**Polynomial interpolations**	
12	**Piecewise interpolation**	
13	**Numerical integration**	
14	**Numerical differnetiation**	
15	**Initial-value problems**	
16	**Final exam**	final exam



Part 1
Chapter 1

Mathematical Modeling, Numerical Methods, and Problem Solving

PowerPoints organized by Dr. Michael R. Gustafson II, Duke University

Chapter Objectives

- Learning how mathematical models can be formulated on the basis of scientific principles to simulate the behavior of a simple physical system.
- Understanding how numerical methods afford a means to generalize solutions in a manner that can be implemented on a digital computer.
- Understanding the different types of conservation laws that lie beneath the models used in the various engineering disciplines and appreciating the difference between steady-state and dynamic solutions of these models.
- Learning about the different types of numerical methods we will cover in this book.

A Simple Mathematical Model

- A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms.
- Models can be represented by a functional relationship between dependent variables, independent variables, parameters, and forcing functions.

Model Function

$$\begin{matrix}\text{Dependent}\\ \text{variable}\end{matrix} = f\left(\begin{matrix}\text{independent}\\ \text{variables}\end{matrix},\ \text{parameters},\ \begin{matrix}\text{forcing}\\ \text{functions}\end{matrix}\right)$$

•*Dependent variable* - a characteristic that usually reflects the behavior or state of the system

•*Independent variables* - dimensions, such as time and space, along which the system's behavior is being determined

•*Parameters* - constants reflective of the system's properties or composition

•*Forcing functions* - external influences acting upon the system

Model Function Example

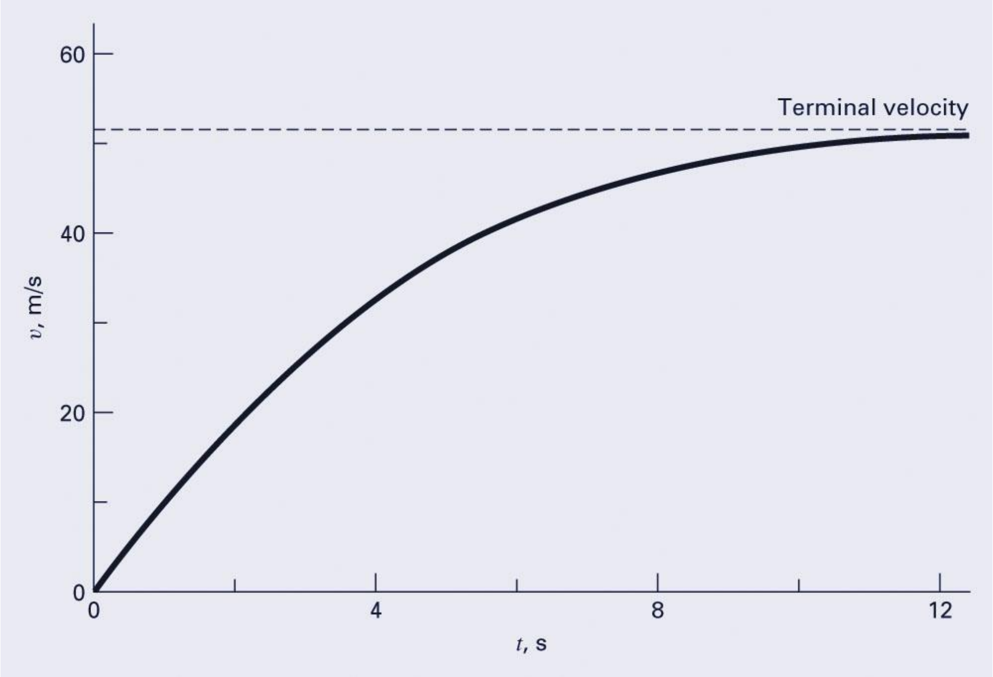
- Assuming a bungee jumper is in mid-flight, an analytical model for the jumper's velocity, accounting for drag, is

$$v(t) = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}} t \right)$$

- Dependent variable - velocity v
- Independent variables - time t
- Parameters - mass m, drag coefficient c_d
- Forcing function - gravitational acceleration g

Model Results

- Using a computer (or a calculator), the model can be used to generate a graphical representation of the system. For example, the graph below represents the velocity of a 68.1 kg jumper, assuming a drag coefficient of 0.25 kg/m



Numerical Modeling

- Some system models will be given as implicit functions or as differential equations - these can be solved either using analytical methods or numerical methods.
- Example - the bungee jumper velocity equation from before is the analytical solution to the differential equation

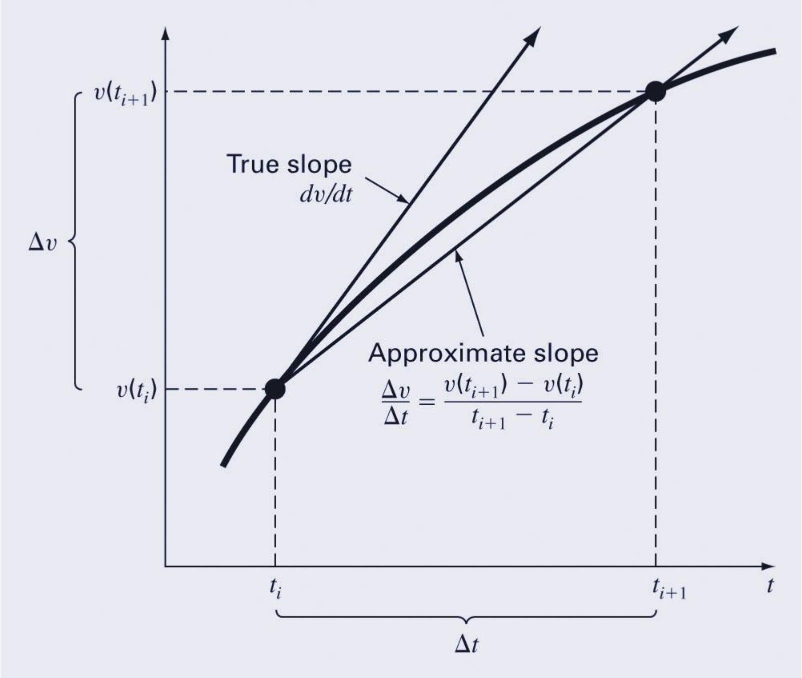
$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

where the change in velocity is determined by the gravitational forces acting on the jumper versus the drag force.

Numerical Methods

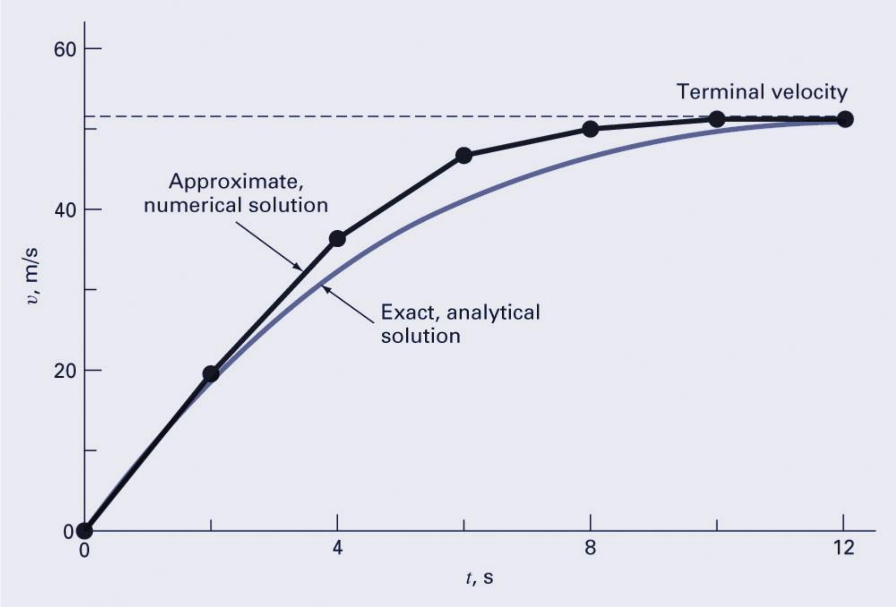
- To solve the problem using a numerical method, note that the time rate of change of velocity can be approximated as:

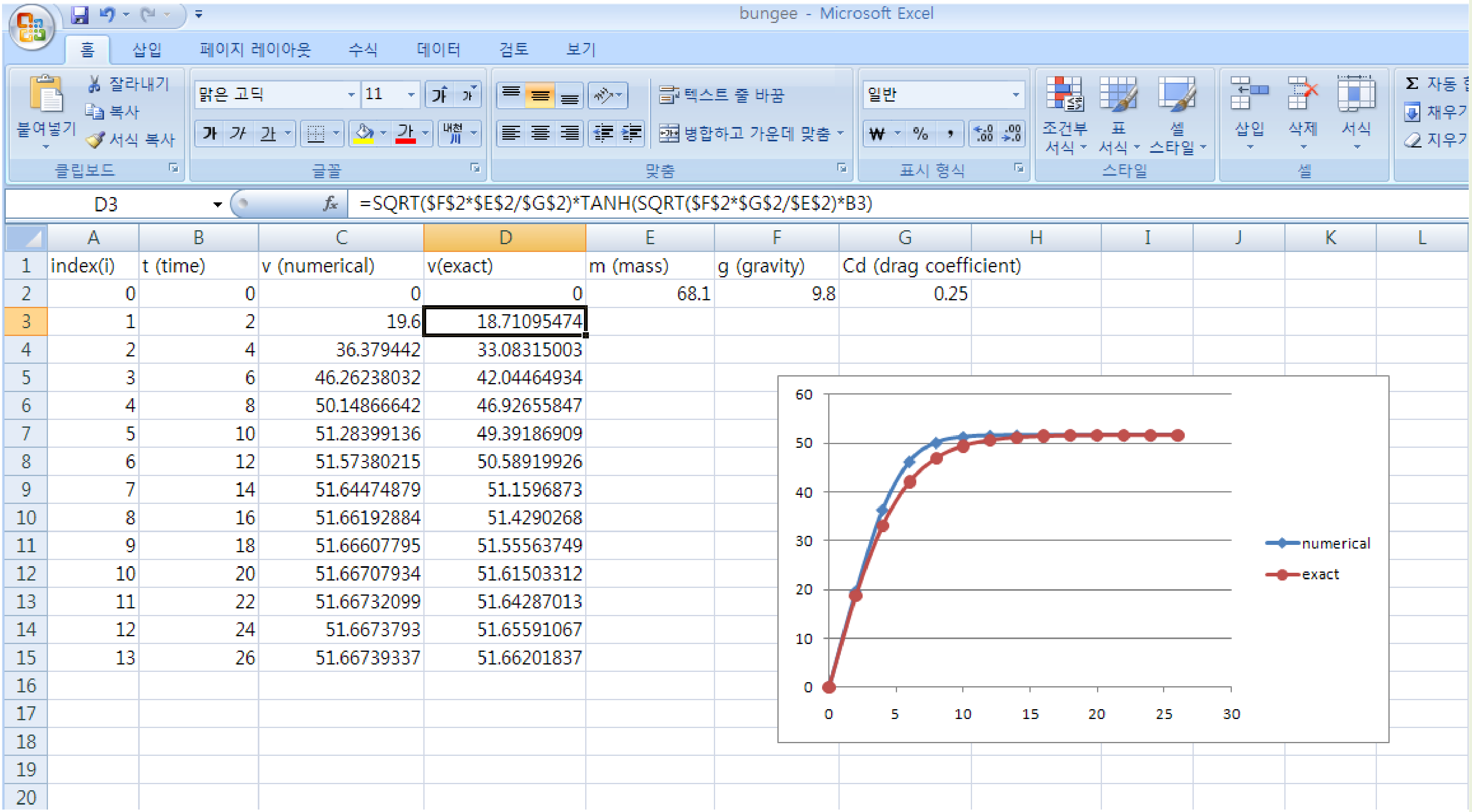
$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$



Numerical Results

- As shown in later chapters, the efficiency and accuracy of numerical methods will depend upon how the method is applied.
- Applying the previous method in 2 s intervals yields:



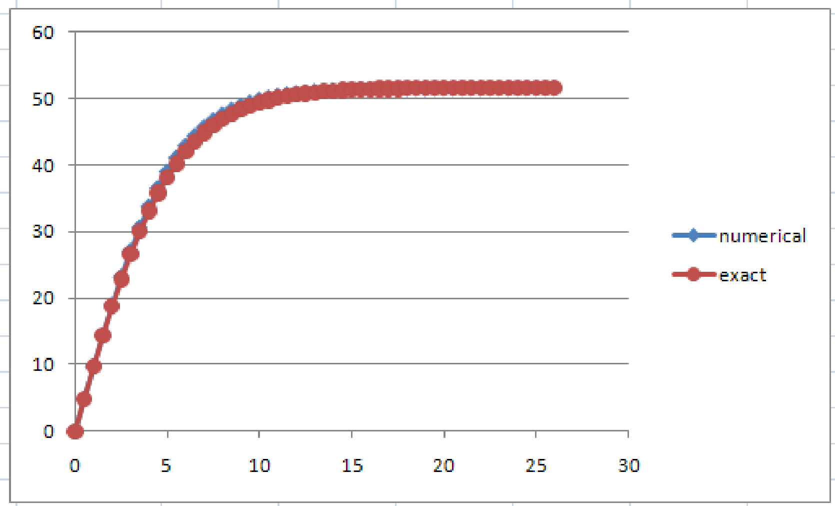


	A	B	C	D	E	F	G
1	index(i)	t (time)	v (numerical)	v(exact)	m (mass)	g (gravity)	Cd (drag coefficient)
2	0	0	0	0	68.1	9.8	0.25
3	1	2	19.6	18.71095474			
4	2	4	36.379442	33.08315003			
5	3	6	46.26238032	42.04464934			
6	4	8	50.14866642	46.92655847			
7	5	10	51.28399136	49.39186909			
8	6	12	51.57380215	50.58919926			
9	7	14	51.64474879	51.1596873			
10	8	16	51.66192884	51.4290268			
11	9	18	51.66607795	51.55563749			
12	10	20	51.66707934	51.61503312			
13	11	22	51.66732099	51.64287013			
14	12	24	51.6673793	51.65591067			
15	13	26	51.66739337	51.66201837			

Time step ($t_{i+1} - t_i$) = 2 second

bungee - Microsoft Excel

	A	B	C	D	E	F	G
1	index(i)	t (time)	v (numerical)	v(exact)	m (mass)	g (gravity)	Cd (drag coefficient)
2	0	0	0	0	68.1	9.8	0.25
3	1	0.5	4.9	4.885362253			
4	2	1	9.755928781	9.684143707			
5	3	1.5	14.48122587	14.31579439			
6	4	2	18.99630314	18.71095474			
7	5	2.5	23.23393249	22.81508219			
8	6	3	27.1430815	26.59022791			
9	7	3.5	30.69075611	30.01501402			
10	8	4	33.86182345	33.08315003			
11	9	4.5	36.65715552	35.800981			
12	10	5	39.09065947	38.18457822			
13	11	5.5	41.18581428	40.25680349			
14	12	6	42.97224729	42.04464934			
15	13	6.5	44.48272079	43.57702436			
16	14	7	45.7507229	44.88304249			
17	15	7.5	46.80870997	45.99079734			
18	16	8	47.68694909	46.92655847			
19	17	8.5	48.41285747	47.71430784			



Time step ($t_{i+1} - t_i$) = 0.5 second

Bases for Numerical Models

- Conservation laws provide the foundation for many model functions.
- Different fields of engineering and science apply these laws to different paradigms within the field.
- Among these laws are:
  - Conservation of mass
  - Conservation of momentum
  - Conservation of charge
  - Conservation of energy

Summary of Numerical Methods

- The book is divided into five categories of numerical methods: