Chapter 4

Energy Equation for a Control Volume (conti.)

- **Throttle**

Abrupt pressure drop, almost negligible change in velocity, ***no change in enthalpy***

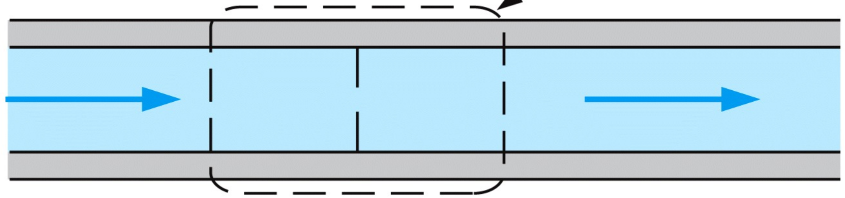


Figure 4.8

Orifice or control valve

Methane at 3 MPa, 300 K, is throttled to 100 kPa. Calculate the exit temperature assuming no changes in the kinetic energy and ideal-gas behavior. Repeat the answer for real-gas behavior.

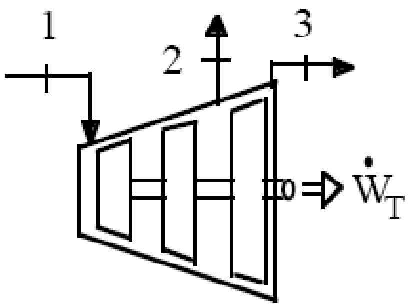
C.V. Throttle (valve, restriction), SSSF, 1 inlet and exit, no q, w

Energy Eq.: $h_i = h_e$ => Ideal gas $T_i = T_e = \mathbf{300\ K}$

Real gas : $\left.\begin{array}{l} h_i = h_e = 598.71 \\ P_e = 0.1\ \text{MPa} \end{array}\right\}$ $\begin{array}{l} \text{Table B.7} \\ T_e = \mathbf{13.85°C}\ (= \mathbf{287\ K}) \end{array}$

- **Turbine**

A steam turbine receives water at 15 MPa, 600°C at a rate of 100 kg/s, shown in Fig. P6.29. In the middle section 20 kg/s is withdrawn at 2 MPa, 350°C, and the rest exits the turbine at 75 kPa, and 95% quality. Assuming no heat transfer and no changes in kinetic energy, find the total turbine power output.



C.V. Turbine SSSF, 1 inlet and 2 exit flows.

Table B.1.3 $h_1 = 3582.3$ kJ/kg, $h_2 = 3137$ kJ/kg

Table B.1.2 : $h_3 = h_f + x_3 h_{fg} = 384.3 + 0.95 \times 2278.6$

$= 2549.1$ kJ/kg

Cont.: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$; Energy: $\dot{m}_1 h_1 = \dot{W}_T + \dot{m}_2 h_2 + \dot{m}_3 h_3$

$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 80$ kg/s , $\dot{W}_T = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = \mathbf{91.565\ MW}$

The Transient Process

- The transient process involve *changes* in the system with time.

- *For simple analysis*, we assume that
  - The CV remains constant relative to the coordinate frame (, but it may involve moving boundaries).
  - The state of the mass at each point in the CV is *uniform* at any instant of time, but it may vary with time.
  - The state of the mass crossing each flow area on the control surface is constant with time, but the flow rates may vary with time.

The Transient Process

- **Continuity Eq.**

$$\boxed{\frac{dm_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e} \longrightarrow \boxed{(m_2 - m_1)_{cv} = \sum m_i - \sum m_e}$$

- **Energy Eq.**

$$\boxed{\frac{dE_{cv}}{dt} = \dot{E}_{in} - \dot{E}_{out} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i h_{tot,i} - \sum \dot{m}_e h_{tot,e}}$$

$$\downarrow$$

$$h_{tot} \equiv h + \frac{1}{2}V^2 + gZ$$

$$\boxed{E_{in} - E_{out} = Q_{cv} - W_{cv} + \sum m_i h_{tot,i} - \sum m_e h_{tot,e}}$$

Example

An initially empty cylinder is filled with air from 20°C, 100 kPa until it is full. Assuming no heat transfer is the final temperature larger, equal to or smaller than 20°C? Does the final T depend on the size of the cylinder?

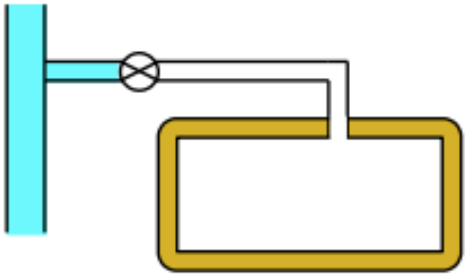
This is a transient problem with no heat transfer and no work. The balance equations for the tank as C.V. become

Continuity Eq.: $m_2 - 0 = m_i$

Energy Eq.: $m_2u_2 - 0 = m_ih_i + Q - W = m_ih_i + 0 - 0$

Final state: $u_2 = h_i$ & $P_2 = P_i$

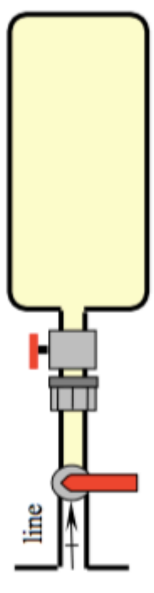
$T_2 > T_i$ and it does not depend on V



Example

A rigid tank of volume 2 m^3 contains air at ambient 100 kPa, 290 K. It is slowly filled with air from a supply line at 1000 kPa and 300 K. After a long time the flow stops and the final temperature in the tank equals the ambient 290 K. How much mass was added to the tank and what was the heat transfer.

Solution:



CV The tank and its content.

Continuity Eq.4.20: $m_2 - m_1 = m_i$

Energy Eq.4.21: $m_2u_2 - m_1u_1 = m_ih_i + {}_1Q_2 - {}_1W_2$

Process: $V = C \quad \Rightarrow \quad {}_1W_2 = \int P\, dV = 0$

State 1 (ideal gas): $m_1 = P_1V_1 / RT_1 = \frac{100 \times 2}{0.287 \times 290} \frac{\text{kPa m}^3}{\text{(kJ/kg-K) K}} = 2.403 \text{ kg}$

When the flow stops we must have line pressure inside the tank.

State 2: $(T_2, P) \Rightarrow \quad m_2 = P_2V_2 / RT_2 = \frac{1000 \times 2}{0.287 \times 290} \frac{\text{kPa m}^3}{\text{(kJ/kg-K) K}} = 24.03 \text{ kg}$

From the continuity equation

$$m_i = m_2 - m_1 = \mathbf{21.627\ kg}$$

Now in the energy equation, $u_2 = u_1$ so we get

$$m_2u_2 - m_1u_1 = m_iu_2 = m_ih_i + {}_1Q_2 - 0$$

so solve for the heat transfer

$$\begin{aligned} {}_1Q_2 &= m_iu_2 - m_ih_i = m_i(u_2 - h_i) = m_i(u_2 - u_i - RT_i) \\ &= m_i[\, C_v(T_2 - T_i) - RT_i] \\ &= 21.627 \text{ kg} [\, 0.717(290 - 300) - 0.287 \times 300] \text{ kJ/kg} \\ &= \mathbf{-2017\ kJ} \end{aligned}$$

Comment: Not only do we have to cool down from the line T, but we also have to remove the energy of the flow work that was coming in.