

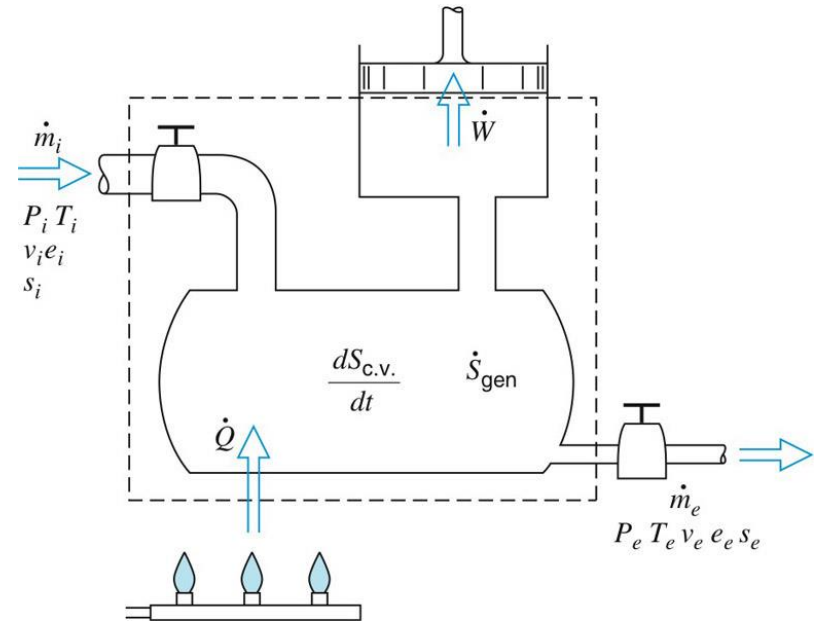
Chapter 7

Entropy Equation for a Control Volume

The Second Law for a Control Volume

- **Rate of change = +in – out + generation**

$$\frac{dS_{cv}}{dt} = \underbrace{\sum \dot{m}_i s_i}_{\text{Entropy flux by mass transfer}} - \underbrace{\sum \dot{m}_e s_e}_{\text{Entropy flux by heat transfer}} + \underbrace{\sum \frac{\dot{Q}_{cv}}{T}}_{\text{Entropy flux by heat transfer}} + \dot{S}_{gen}$$



$$\frac{dS_{cv}}{dt} \geq \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T}$$

- ✓ equality: internally reversible
- ✓ inequality: internally irreversible

* When the heat transfer is distributed over the control surface,

$$\sum \frac{\dot{Q}_{cv}}{T} = \int \frac{d\dot{Q}}{T} = \int_{\text{surface}} \frac{(\dot{Q}/A)_{\text{local}}}{T} dA$$

The Steady-State Process and the Transient Process

- For a steady process,

$$\frac{dS_{cv}}{dt} = 0 \quad \rightarrow \quad \sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum_{cs} \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

- For a steady process with a single inlet and a single outlet,

$$\dot{m}(s_e - s_i) = \sum_{cs} \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

$$s_e = s_i + \sum_{cs} \frac{q_{cv}}{T} + s_{gen} \xrightarrow{+adiabatic} s_e = s_i + s_{gen} \xrightarrow{+reversible} s_e = s_i$$

- For a transient process,

$$\frac{d}{dt}(ms)_{cv} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} + \dot{S}_{gen}$$

- From integration with respect to time t ,

$$(m_2 s_2 - m_1 s_1)_{cv} = \sum m_i s_i - \sum m_e s_e + \int_0^t \sum \frac{\dot{Q}_{cv}}{T} dt + {}_1S_2_{gen}$$

- Besides, when the temperature is *uniform* throughout the control volume at any moment,

$$\int_0^t \sum \frac{\dot{Q}_{cv}}{T} dt = \int_0^t \frac{1}{T} \sum \dot{Q}_{cv} dt = \int_0^t \frac{\dot{Q}_{cv}}{T} dt$$

$$(m_2 s_2 - m_1 s_1)_{cv} = \sum m_i s_i - \sum m_e s_e + \int_0^t \frac{\dot{Q}_{cv}}{T} dt + {}_1S_2_{gen}$$

The Steady-State Single-Flow Process

- Now we derive the expression for the work in a **steady-state, single flow process**.

$$\text{Energy Eq.: } q + h_i + \frac{1}{2}V_i^2 + gZ_i = h_e + \frac{1}{2}V_e^2 + gZ_e + w \leftarrow$$

$$\text{Entropy Eq.: } s_e = s_i + \int \frac{\delta q}{T} + s_{gen}$$

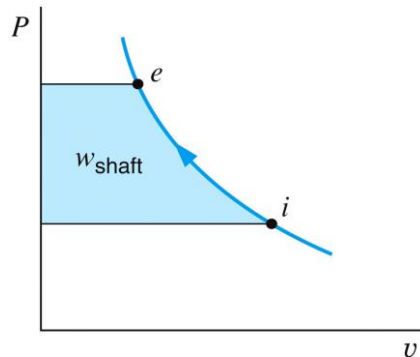
$$\delta s_{gen} + \frac{\delta q}{T} = ds \quad \Rightarrow \quad \delta q = Tds - T\delta s_{gen} = dh - vdP - T\delta s_{gen}$$

$$\begin{aligned} \Rightarrow q &= \int_i^e \delta q = \int_i^e dh - \int_i^e vdP - \int_i^e T\delta s_{gen} \\ &= h_e - h_i - \int_i^e vdP - \int_i^e T\delta s_{gen} \end{aligned}$$

$$w_{shaft} = -\int_i^e vdP + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e) - \int_i^e T\delta s_{gen}$$

$$w_{shaft} = -\int_i^e v dP + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e) - \int_i^e T \delta s_{gen}$$

- ✓ The above expression is useful to account for the *work of turbines, compressors, and pumps*.
 - ✓ The maximum work is produced when the entropy generation is zero.
 - ✓ For a reversible process, work can be extracted from decrease in pressure, kinetic energy or potential energy.
 - ✓ A large amount of work is involved when the specific volume is large.
 - ✓ In a real fluid flow without shaft work or change in the kinetic energy and the potential energy, pressure decreases due to friction.
- When the process is reversible and steady without change in ke and pe, the above expression reduces to



$$w_{shaft} = -\int_i^e v dP$$

- When the process is *reversible* and the fluid is *incompressible*,

$$w_{shaft} = -v(P_e - P_i) + \frac{1}{2}(V_i^2 - V_e^2) + g(Z_i - Z_e)$$

- For zero work,

$$vP_i + \frac{1}{2}V_i^2 + gZ_i = vP_e + \frac{1}{2}V_e^2 + gZ_e \quad (\text{Bernoulli equation})$$

- * For a steady, reversible polytropic process for an ideal gas with no change in kinetic and potential energies, we have the following relation.

$$w = -\int_i^e v dP \quad \text{and} \quad Pv^n = \text{constant} = C^n$$

$$w = -\int_i^e v dP = -C \int_i^e \frac{dP}{P^{1/n}} = -\frac{n}{n-1} (p_e v_e - p_i v_i) = -\frac{nR}{n-1} (T_e - T_i) \quad (n \neq 1)$$

or

$$w = -\int_i^e v dP = -\text{constant} \int_i^e \frac{dP}{P} = -P_i v_i \ln \frac{P_e}{P_i} \quad (n=1)$$

Principle of the Increase of Entropy

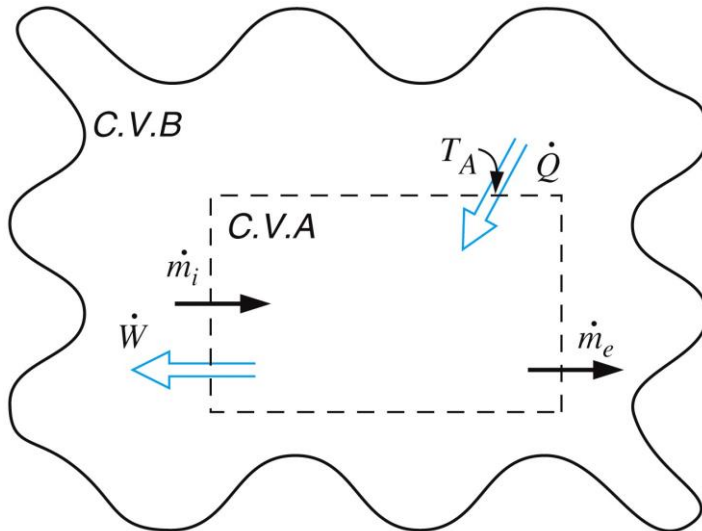


Figure 7.9
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$$\frac{dS_{cv_A}}{dt} = \dot{m}_i s_i - \dot{m}_e s_e + \frac{\dot{Q}_{cv}}{T_A} + \dot{S}_{gen_A}$$

$$\frac{dS_{cv_B}}{dt} = -\dot{m}_i s_i + \dot{m}_e s_e - \frac{\dot{Q}_{cv}}{T_A} + \dot{S}_{gen_B}$$

$$\frac{dS_{net}}{dt} = \frac{dS_{cv_A}}{dt} + \frac{dS_{cv_B}}{dt} = \dot{S}_{gen_A} + \dot{S}_{gen_B} \geq 0$$

- In typical mechanical devices with multiple valves used to control the fluid flow, they introduce irreversible throttling processes besides having an irreversible mixing and possible heat transfer over finite temperature differences.