HW1 (Numerical Analysis)

Due: 2017/04/07

1. Locate the first nontrivial root of sin(x) = x^2 where x is in radians. Use a graphical technique and bisection method with the initial interval from 0.5 to 1. Perform the computation until $\boldsymbol{\varepsilon}_a$ is less than $\boldsymbol{\varepsilon}_s$ = 2%.

i	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$\lvert\varepsilon_a\rvert$
1	0.5	0.229426	1	−0.158529	0.75	0.1191388	
2	0.75	0.119139	1	−0.158529	0.875	0.0019185	14.29%
3	0.875	0.001919	1	−0.158529	0.9375	−0.0728251	6.67%
4	0.875	0.001919	0.9375	−0.0728251	0.90625	−0.0340924	3.45%
5	0.875	0.001919	0.90625	−0.0340924	0.890625	−0.0157479	1.75%

2. Employ fixed-point iteration to locate the root of f(x) = sin(sqrt(x))-x. Use an initial guess of x_0 = 0.5 and iterate until $\boldsymbol{\varepsilon}_a$ < 0.01%. Verify that the process is linearly convergent.

iteration	x_i	$\lvert\varepsilon_a\rvert$
0	0.500000	
1	0.649637	23.0339%
2	0.721524	9.9632%
3	0.750901	3.9123%
4	0.762097	1.4691%
5	0.766248	0.5418%
6	0.767772	0.1984%
7	0.768329	0.0725%
8	0.768532	0.0265%
9	0.768606	0.0097%

3. Employ the Newton-Raphson method to determine a real root for f(x) = -2 + 6x -4x^2 + 0.5x^3, using an initial guess of (a) 4.5 and (b) 4.43. Discuss and use graphical and analytical methods to explain any peculiarities in your results.

iteration	x_i	$f(x_i)$	$f'(x_i)$	$\lvert\varepsilon_a\rvert$
0	4.5	−10.4375	0.375	
1	32.333330	12911.57	1315.5	86.082%
2	22.518380	3814.08	586.469	43.586%
3	16.014910	1121.912	262.5968	40.609%
4	11.742540	326.4795	118.8906	36.384%

5	8.996489	92.30526	55.43331	30.524%
6	7.331330	24.01802	27.97196	22.713%
7	6.472684	4.842169	17.06199	13.266%
8	6.188886	0.448386	13.94237	4.586%
9	6.156726	0.005448	13.6041	0.522%
10	6.156325	8.39E–07	13.59991	0.007%

(b) Using an initial guess of 4.43, the iterations proceed as

iteration	x_i	$f(x_i)$	$f'(x_i)$	$\lvert\varepsilon_a\rvert$
0	4.43	-10.4504	-0.00265	
1	-3939.13	-3.1E+10	23306693	100.112%
2	-2625.2	-9.1E+09	10358532	50.051%
3	-1749.25	-2.7E+09	4603793	50.076%
4	-1165.28	-8E+08	2046132	50.114%
5	-775.964	-2.4E+08	909393.5	50.171%
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21	0.325261	-0.45441	3.556607	105.549%
22	0.453025	-0.05629	2.683645	28.203%
23	0.474	-0.00146	2.545015	4.425%
24	0.474572	-1.1E-06	2.541252	0.121%
25	0.474572	-5.9E-13	2.541249	0.000%

4. **[MATLAB]** Solve example 4.3 (approximation of a function with a Taylor series expansion) using MATLAB. Show your MATLAB code and the result in a table. Plot a $|\varepsilon_t|$ vs **n** graph also using MATLAB.

5. **[MATLAB]** Use the false position method to locate the root of the function: $F(x) = x^{10} - 1$. Show your MATLAB code and the calculated root x_r and approximate percent relative error $|\varepsilon_a|$ in a table. Perform your iterative calculation until n = 6.