Chapter 3 (conti.)

First Law of Thermodynamics
and Energy Equation

Work done at the moving boundary of a simple compressible system

- In a quasi-equilibrium process,

$$\boxed{\delta W_b = FdL = PAdL = PdV}$$

Boundary work

cf. non-equilibrium process: $\delta W = (p_{ext}A)dL = p_{ext}DV$

- ex) in a compression process,
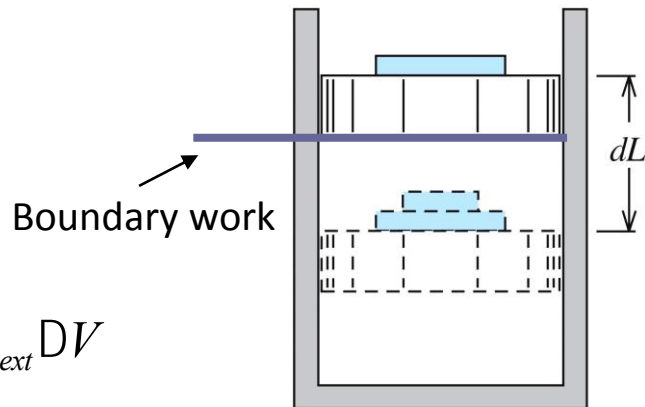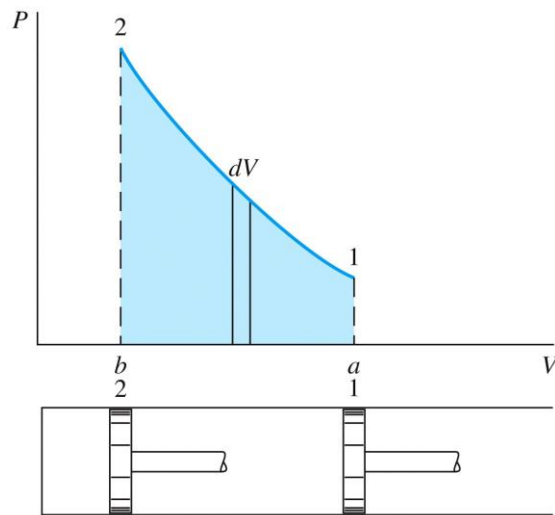
$$_1W_2 = \int_1^2 \delta W = \int_1^2 P\,dV$$

= the **area** of the shaded region under the P–V curve (a–1–2–b–a)

- As discussed earlier, work is a path function, and δW is an inexact differential. In other words, work is not a thermodynamic property

$$_1W_2 = \int_1^2 \delta W \neq W_2 - W_1$$

cf. $$_1V_2 = \int_1^2 dV = V_2 - V_1$$

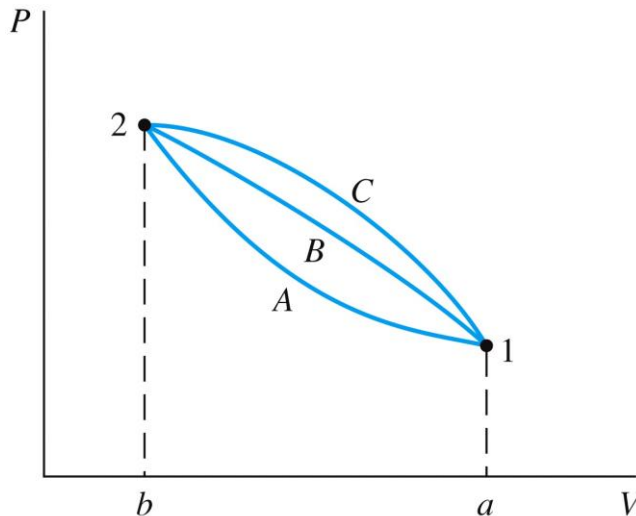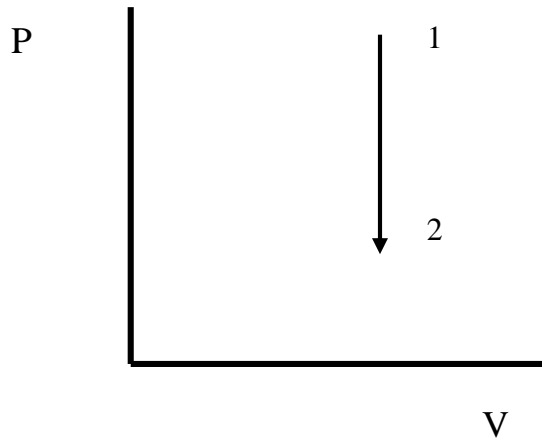The cyclic integral $\oint \delta W$ can be nonzero.

cf. $\oint dV = 0$

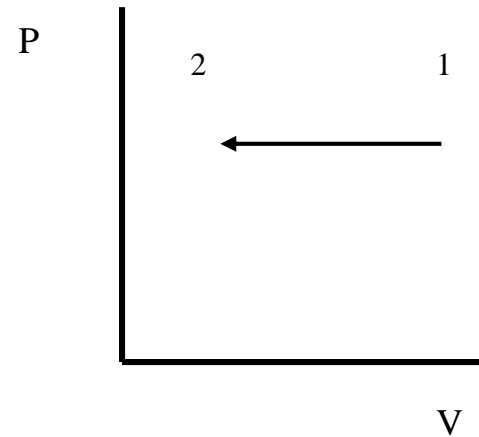# Work by the moving boundary in special cases

If the volume is held constant ($dV = 0$), then the boundary work equation becomes

P

1

2

V

P-V diagram for V = Constant

$$W_b = \int_1^2 P\,dV = 0$$

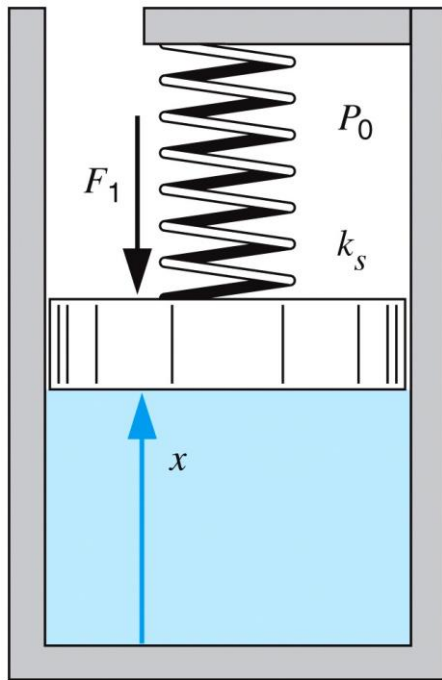If the Pressure is held constant ($dP = 0$), then the boundary work equation becomes

P

2   1

V

P-V diagram for P = Constant

$$W_b = \int_1^2 P\,dV = P\int_1^2 dV$$

$$= P(V_2 - V_1)$$

Example 3.4

Consider a slightly different piston/cylinder arrangement, as shown in Fig. 3.10. In this example, the piston is loaded with a mass m_p the outside atmosphere P_0, a linear spring and a single point force F_1. The piston traps the gas inside with a pressure P. A force balance on the piston in the direction of motion yields

Figure 3.10
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$$m_p a = 0 = \mathring{a} F_{\uparrow} - \mathring{a} F_{\downarrow}$$

$$\mathring{a} F_{\uparrow} = \mathring{a} F_{\downarrow}$$

$$\mathring{a} F_{\uparrow} = PA$$

$$\mathring{a} F_{\downarrow} = m_p g + P_o A + k_s (x - x_0) + F_1$$

$$P = \left\{ m_p g + P_o A + k_s (x - x_0) + F_1 \right\} / A$$

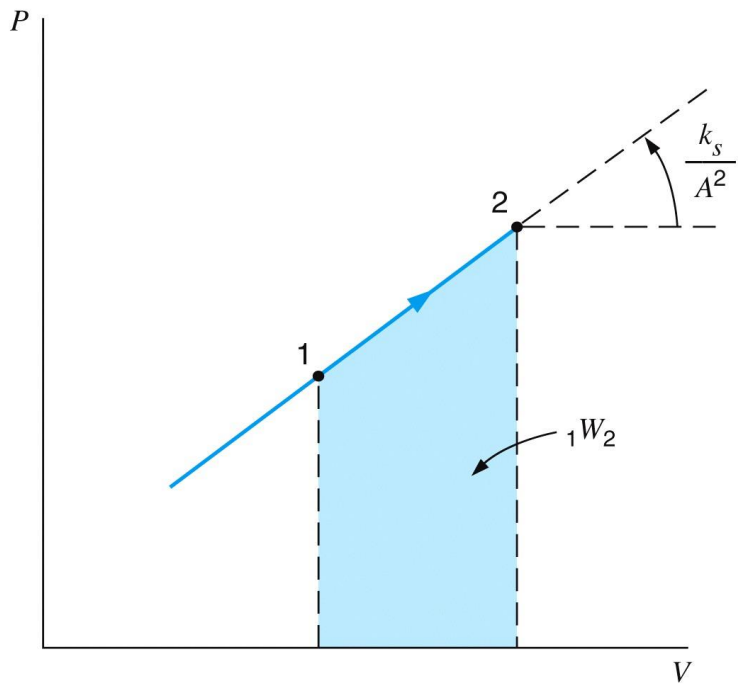$$P = P_o + \frac{m_p g}{A} + \frac{F_1}{A} + \frac{k_s}{A^2}(V - V_0) = C_1 + C_2 V$$

slope

Figure 3.11
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$$_1W_2 = \int\limits_{1}^{2} PdV = \frac{1}{2}\left(P_1 + P_2\right)\left(V_2 - V_1\right)$$

Polytropic process

$$PV^n = const$$

- The polytropic exponent n indicates the type of the process

Process	Exponent n
Constant pressure (or isobaric) process	0
Constant volume (or isochoric) process	±inf
Constant T (Isothermal) process of ideal gas	1

$$PV^n = const$$

$$P_1V_1^n = P_2V_2^n = const$$

$$P = \frac{const}{V^n} = \frac{P_1V_1^n}{V^n} = \frac{P_2V_2^n}{V^n}$$

Integration in a polytropic process

- When $n \neq 1$,

$$\int_1^2 P\,dV = const\int_1^2 \frac{dV}{V^n} = const\left(\frac{V^{-n+1}}{-n+1}\right)\Bigg|_1^2$$

$$= \frac{const}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right) = \frac{P_2 V_2^n V_2^{1-n} - P_1 V_1^n V_1^{1-n}}{1-n}$$

$$= \frac{P_2 V_2 - P_1 V_1}{1-n} \Rightarrow \frac{mR(T_2 - T_1)}{1-n} \text{ (ideal gas)}$$

- When n = 1,

$$\int_1^2 P\,dV = P_1 V_1 \int_1^2 \frac{dV}{V} = P_1 V_1 \ln\frac{V_2}{V_1}$$

Work in a Polytropic Process

Example 3.6

An ideal gas in a piston cylinder starts out with 200 kPa, 0.04 m^3. Let us look at 4 different possible processes and find the work for each case.

Solution

a. Process: $P = C$, Heat the gas to $V_2 = 0.1$ m^3

$$_1W_2 = \int_1^2 P \, dV = P \int dV = P \,(V_2 - V_1)$$

$$= 200 \text{ kPa} \times (0.1 - 0.04) \text{ m}^3 = 12.0 \text{ kJ}$$

b. Process: $T = C$, Heat the gas to $V_2 = 0.1$ m^3

Ideal gas: $PV = m\,RT = C$ so polytropic with $n = 1$

$$_1W_2 = \int_1^2 P \, dV = P_1 V_1 \ln \frac{V_2}{V_1}$$

$$= 200 \text{ kPa} \times 0.04 \text{ m}^3 \times \ln \frac{0.1}{0.04} = 7.33 \text{ kJ}$$

To do the process the mass (load) on piston varies

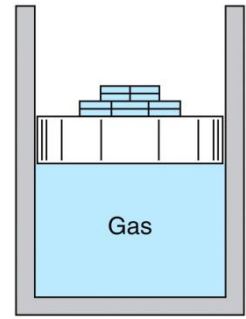# Work in a Polytropic Process

Example 3.6 continued

c. Process: Polytropic with $n = 1.3$, Increase volume to $V_2 = 0.1 \text{ m}^3$

$$P_2 = P_1 \, (V_1 / V_2)^{1.3} = 200 \text{ kPa} \times (0.04 / 0.1)^{1.3} = 60.77 \text{ kPa}$$

$$_1W_2 = \frac{1}{1 - n} \left(P_2 V_2 - P_1 V_1 \right)$$

$$= \frac{1}{1 - 1.3} (60.77 \times 0.1 - 200 \times 0.04) \text{ kPa m}^3$$

$$= 6.41 \text{ kJ}$$

d. Process: $V = C$, Cool the gas to $P_2 = 100 \text{ kPa}$

$$_1W_2 = \int_1^2 P \, dV = 0$$

Comment:
To do the process (c) the mass (load) on piston varies
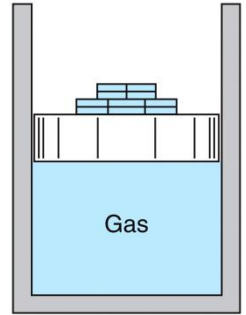To do the process (d) the piston is locked in place

Figure 3.14
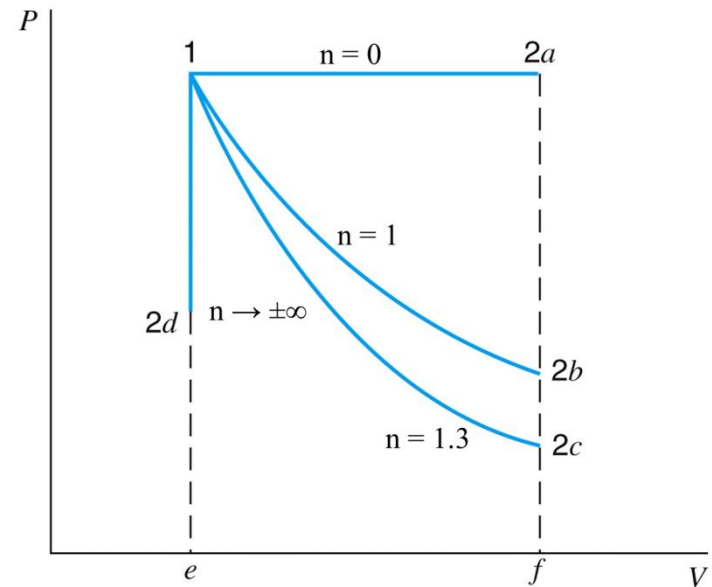© John Wiley & Sons, Inc. All rights reserved.

Figure 3.15
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Heat

- ***Heat (Q)*** is defined as the form of energy that is <u>transferred across the boundary</u> of a system at a given temperature to another system (or the surroundings) at a lower temperature <u>by virtue of the temperature difference between the two system</u>.

    ✓ Transient phenomenon (A body never contains heat.)

    ✓ A path function that has an inexact differential.
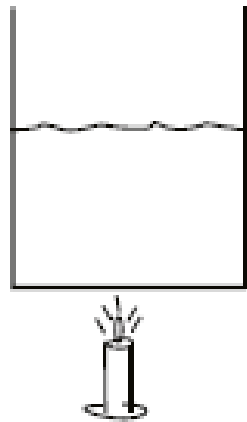
$$\int_1^2 dQ = {}_1Q_2$$

    ✓ Unit: joule, calorie (heat needed to raise 1 g of water from 14.5°C to 15.5°C, 1 cal = 4.2 J)

- ***Adiabatic process*** : a process with no heat transfer (Q=0)

- Specific heat transfer

$$q \circ \frac{Q}{m}$$
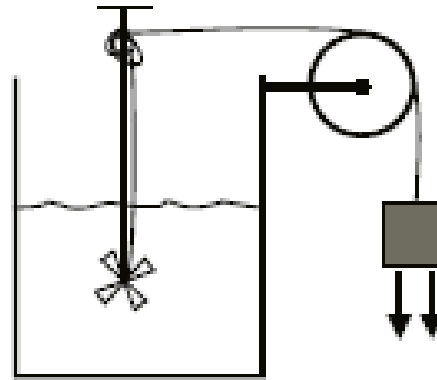
Equivalence of work and heat

Joule : it is possible to raise the temperature of H_2O

a) with only heat   b) with only work

Both transient phenomena
Both boundary phenomena
Both path function and inexact differential
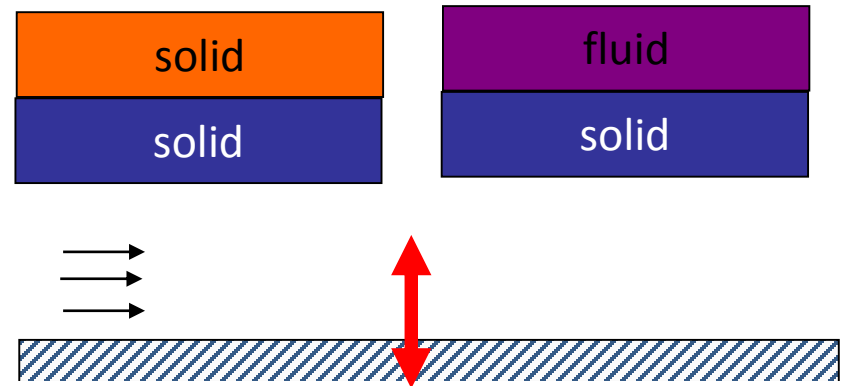
Heat Transfer Modes

- **Conductive heat transfer**

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$

- **Convective heat transfer**

$$\dot{Q}_{conv} = hA\Delta T$$

- **Radiative heat transfer**

$$\dot{Q}_{rad} = \varepsilon A\sigma T^4$$

solid

solid

fluid

solid

Internal Energy – a Thermodynamic Property

- U : internal energy (extensive property)

- $u = U/m$: specific internal energy (intensive property)

- **Internal energy** is <u>a thermodynamic property</u> that can substitute for an independent property of a pure substance. In the case of a pure substance, therefore, the internal energy can be obtained from two independent thermodynamic properties. (refer to table B.1.1 and B.1.2)

- In the steam tables, the internal energy values are obtained from calculation <u>relative to a reference</u>: zero for saturated liquid at the triple-point temperature.

- In the liquid-vapor saturation region,

$$U = U_{liq} + U_{vap}, \quad mu = m_{liq}u_f + m_{vap}u_g$$

$$u = (1 - x)u_f + xu_g$$

$$u = u_f + xu_{fg}$$

The Internal Energy

Example 3.8

Determine the missing property (P, T, v, x) for water at each of the following states:

    a. $T = 300^oC$, u = 2780 kJ/kg ;   **b.** $P = 200$ kPa, u = 2000 kJ/kg

Solution _____

a) Table B.1.1 at 300^oC: $u > u_g = 2563.0$ kJ/kg, so superheated vapor,

                x is undefined,

Table B.1.3 between 1600 and 1800 kPa at 300^oC:
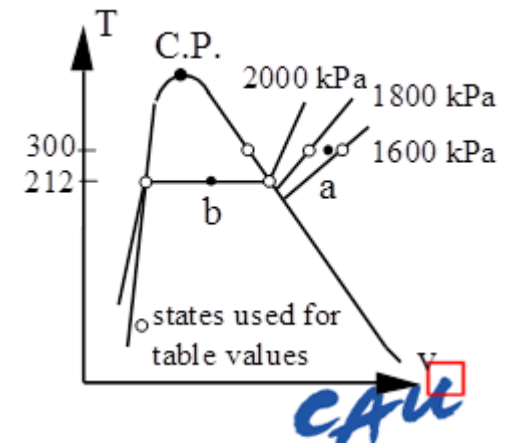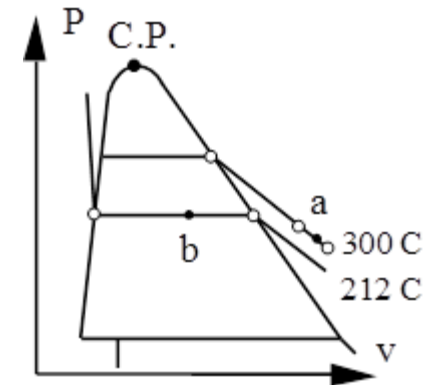
    1600 kPa, 300^oC: u = 2781.03 kJ/kg;   v = 0.15862 m^3/kg

    1800 kPa, 300^oC: u = 2776.83 kJ/kg;   v = 0.14021 m^3/kg

linear interpolation:

$$P = 1600 + 200 \; \frac{2780 - 2781.03}{2776.83 - 2781.03} = 1600 + 200 \times 0.24524$$

$$= 1648 \text{ kPa}$$

$$v = 0.15862 + (0.14021 - 0.15862) \times 0.24524 = 0.1542 \text{ } m^3/kg$$

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The Internal Energy

Example 3.8 continued
Determine the missing property (P, T, v, x) for water at each of the following states:

     a. $T = 300^\circ C$, u = 2780 kJ/kg ;   **b.** $P = 200$ kPa, u = 2000 kJ/kg

Solution
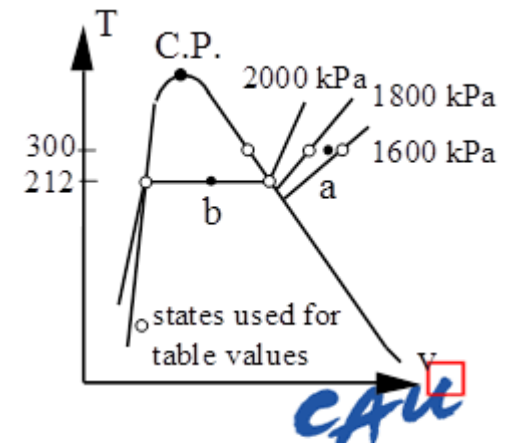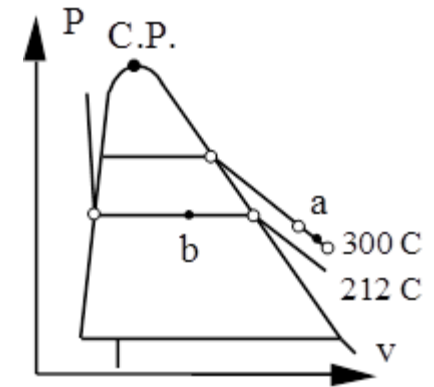
b) Table B.1.2 at 2000 kPa:

    $u_f = 906.4 < u < u_g = 2600.3$ kJ/kg $\Rightarrow$ two-phase

    $u = 2000 = u_f + x\, u_{fg} = 906.4 + x \times 1693.8$

             $\Rightarrow$ $x = 0.6456$

    $v = v_f + x\, v_{fg} = 0.001\ 177 + 0.6456 \times 0.098\ 45$

        $= \mathbf{0.064\ 74\ m^3/kg}$

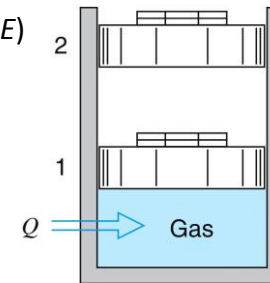Thermodynamics I   School of Mechanical Engineering

Enthalpy – a Thermodynamic Property

- $H \equiv U + PV$ (extensive property)

- $h = H/m = u + pv$: specific enthalpy (intensive property)

- For volumetric expansion of a gas in a *constant-pressure* quasi-equilibrium process, $\delta Q = dU + \delta W$ (Negligible *KE* & *PE*)

$$\delta Q = dU + PdV$$

$$\delta Q = d(U + PV) = dH$$

$$_1Q_2 = H_2 - H_1$$

- In the steam tables, the enthalpy values are obtained from calculation <u>relative to a reference</u>.

- In the liquid-vapor saturation region,

$$H = H_{liq} + H_{vap} \quad , \quad mh = m_{liq}h_f + m_{vap}h_g$$

$$h = (1 - x)h_f + xh_g$$

$$h = h_f + xh_{fg}$$