HW 4 (Numerical Analysis)

Due: 2017/06/13

1. Evaluate the following integral:

$$\int_0^4 (1-e^{-2x})dx$$

(a) Analytically, (b) single application of the trapezoidal rule, (c) composite trapezoidal rule with n=2 and 4, (d) single application of Simpson's 1/3 rule, (e) composite Simpson's 1/3 rule with n=4, (f) Simpson's 3/8 rule. For each of the numerical estimates (b) through (g), determine the true percent relative error based on (a).

16.2 (a) The analytical solution can be evaluated as

$$\int_0^4 (1-e^{-2x})\,dx = \left[x + 0.5e^{-2x}\right]_0^4 = 4 + 0.5e^{-2(4)} - 0 - 0.5e^{-2(0)} = 3.500167731$$

(b) single application of the trapezoidal rule

$$(4-0)\frac{0+0.999665}{2} = 1.99329 \quad (\varepsilon_t = 42.88\%)$$

(c) composite trapezoidal rule

<u>$n = 2$:</u>

$$(4-0)\frac{0+2(0.981684)+0.999665}{4} = 2.96303 \quad (\varepsilon_t = 15.35\%)$$

<u>$n = 4$:</u>

$$(4-0)\frac{0+2(0.86466+0.981684+0.99752)+0.999665}{8} = 3.3437 \quad (\varepsilon_t = 4.47\%)$$

(d) single application of Simpson's 1/3 rule

$$(4-0)\frac{0+4(0.981684)+0.999665}{6} = 3.28427 \quad (\varepsilon_t = 6.17\%)$$

(e) composite Simpson's 1/3 rule ($n = 4$)

$$(4-0)\frac{0+4(0.86466+0.99752)+2(0.981684)+0.999665}{12} = 3.47059 \quad (\varepsilon_t = 0.84\%)$$

(f) Simpson's 3/8 rule.

$$(4-0)\frac{0+3(0.930517+0.995172)+0.999665}{8} = 3.388365 \quad (\varepsilon_t = 3.19\%)$$

2. Solve the following problem over the interval from t=0 to 1 using a step size of 0.25 where y(0) = 1. Display all your results on the same graph.

$$\frac{dy}{dt} = (1 + 2t)\sqrt{y}$$

(a) Analytically.
(b) Using Euler's method.
(c) Using Heun's method without iteration.
(d) Using the fourth-order RK method. (optional)

18.2 **(a)** The analytical solution can be derived by the separation of variables,

$$\int \frac{dy}{\sqrt{y}} = \int 1 + 2x \, dx$$

The integrals can be evaluated to give,

$$2\sqrt{y} = x + x^2 + C$$

Substituting the initial conditions yields $C = 2$. Substituting this value and rearranging gives

$$y = \left(\frac{x^2 + x + 2}{2} \right)^2$$

Some selected value can be computed as

x	*y*
0	**1**
0.25	**1.336914**
0.5	**1.890625**
0.75	**2.743164**
1	**4**

(b) Euler's method:

$$y(0.25) = y(0) + f(0,1)h$$

$$f(0,1) = (1 + 2(0))\sqrt{1} = 1$$

$$y(0.25) = 1 + 1(0.25) = 1.25$$

$$y(0.5) = y(0.25) + f(0.25,1.25)0.25$$

$$f(0.25,1.25) = (1 + 2(0.25))\sqrt{1.25} = 1.67705$$

$$y(0.5) = 1.25 + 1.67705(0.25) = 1.66926$$

The remaining steps can be implemented and summarized as

x	*y*	*dy/dx*
0	**1**	1
0.25	**1.25**	1.67705
0.5	**1.66926**	2.584
0.75	**2.31526**	3.804
1	**3.26626**	5.42184

(c) Heun's method:

Predictor:

Predictor:

$$k_1 = (1 + 2(0))\sqrt{1} = 1$$

$$y(0.25) = 1 + 1(0.25) = 1.25$$

$$k_2 = (1 + 2(0.25))\sqrt{1.25} = 1.6771$$

Corrector:

$$y(0.25) = 1 + \frac{1 + 1.6771}{2} 0.25 = 1.33463$$

The remaining steps can be implemented and summarized as

x	y	k_1	x_e	y_e	k_2	dy/dx
0	**1**	1.0000	0.25	1.25	1.6771	1.3385
0.25	**1.33463**	1.7329	0.5	1.76785	2.6592	2.1961
0.5	**1.88364**	2.7449	0.75	2.56987	4.0077	3.3763
0.75	**2.72772**	4.1290	1	3.75996	5.8172	4.9731
1	**3.97099**					

(e) RK4

x	y	k_1	x_m	y_m	k_2	x_m	y_m	k_3	x_e	y_e	k_4	ϕ
0	**1.0000**	1	0.125	1.1250	1.32583	0.125	1.1657	1.34961	0.25	1.3374	1.73469	1.3476
0.25	**1.3369**	1.73436	0.375	1.5537	2.18133	0.375	1.6096	2.2202	0.5	1.8919	2.75096	2.2147
0.5	**1.8906**	2.74997	0.625	2.2343	3.36322	0.625	2.3110	3.42043	0.75	2.7457	4.14253	3.4100
0.75	**2.7431**	4.14056	0.875	3.2606	4.96574	0.875	3.3638	5.04368	1	4.0040	6.00299	5.0271
1	**3.9998**