Chapter 5 (conti.)

The Classical Second Law of Thermodynamics

The Carnot Cycle

- From the 2nd law, it is impossible to build a 100% efficient heat engine.
- ***The Carnot cycle is the most efficient cycle that can operate between two constant temperature reservoirs.***
- The Carnot cycle consists of four basic processes.

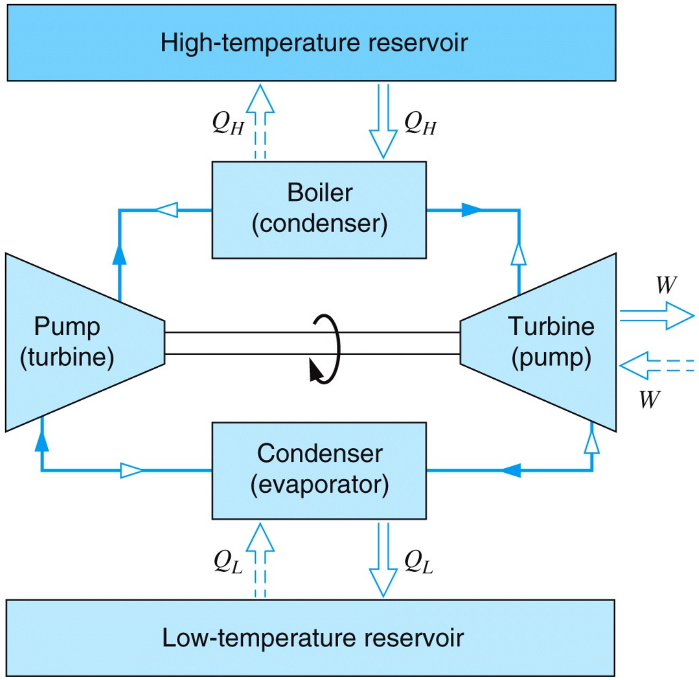
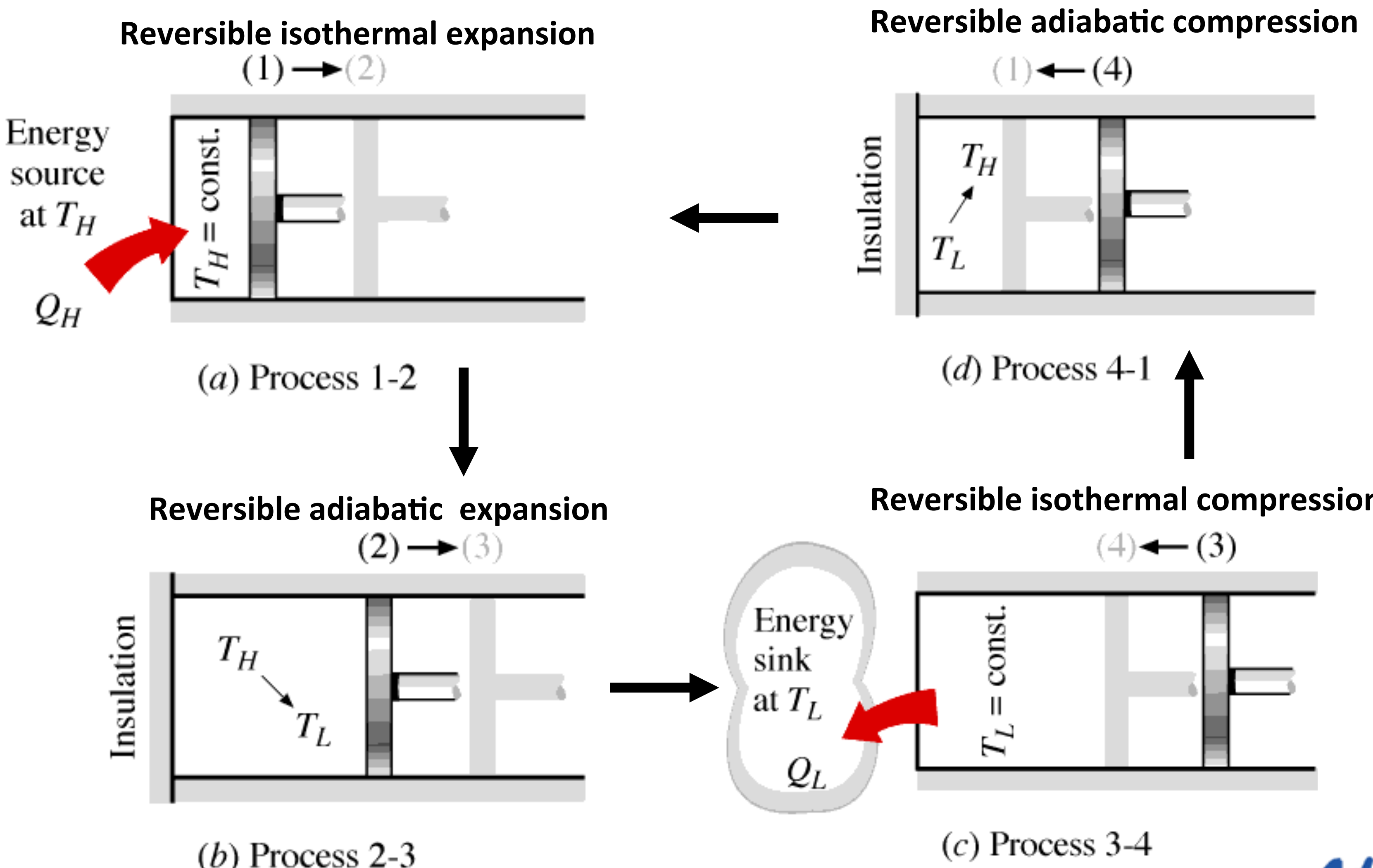


Figure 5.18

1. **Reversible isothermal expansion**
2. **Reversible adiabatic expansion**
3. **Reversible isothermal compression**
4. **Reversible adiabatic compression**

The Carnot Cycle in a Piston/Cylinder System



The Carnot Cycle in a *P-v* diagram

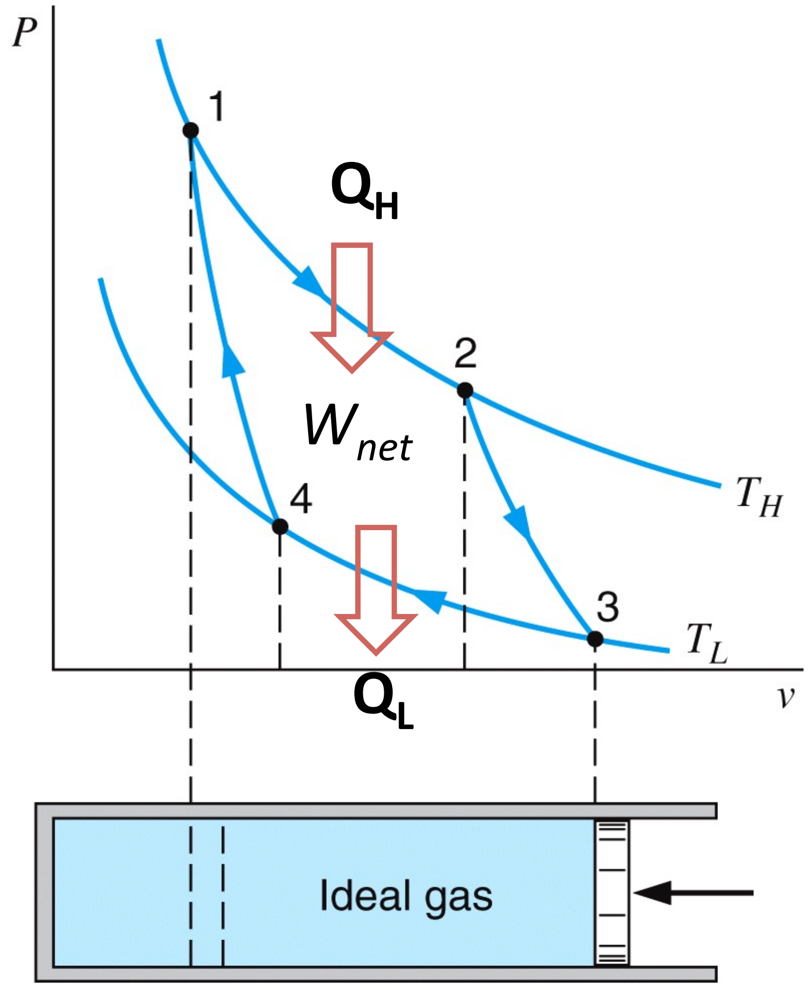


Figure 5.24

In the case of an *ideal gas*,

$$\delta w = Pdv = \frac{RT}{v}dv$$

$$du = C_{v0}dT$$

$$\therefore \delta q = du + \delta w = C_{v0}dT + \frac{RT}{v}dv$$

$$1 \rightarrow 2 : q_H = {}_1q_2 = 0 + RT_H \ln\frac{v_2}{v_1} \qquad \text{Eq. (1)}$$

$$2 \rightarrow 3 : 0 = \int_{T_H}^{T_L} \frac{C_{v0}}{T}dT + R\ln\frac{v_3}{v_2} \qquad \text{Eq. (2)}$$

$$3 \rightarrow 4 : q_L = -{}_3q_4 = -0 - RT_L \ln\frac{v_4}{v_3} = RT_L \ln\frac{v_3}{v_4} \qquad \text{Eq. (3)}$$

$$4 \rightarrow 1 : 0 = \int_{T_L}^{T_H} \frac{C_{v0}}{T}dT + R\ln\frac{v_1}{v_4} \qquad \text{Eq. (4)}$$

From Eq. (2) and (4),

$$\int_{T_L}^{T_H} \frac{C_{v0}}{T} dT = R \ln \frac{v_3}{v_2} = -R \ln \frac{v_1}{v_4}$$

$$\therefore \frac{v_3}{v_2} = \frac{v_4}{v_1}, \quad \text{or} \quad \frac{v_3}{v_4} = \frac{v_2}{v_1}$$

From Eq. (1) and (3),

$$\boxed{\frac{\boldsymbol{q_H}}{\boldsymbol{q_L}} = \frac{RT_H \ln \dfrac{v_2}{v_1}}{RT_L \ln \dfrac{v_3}{v_4}} = \frac{\boldsymbol{T_H}}{\boldsymbol{T_L}}}$$

- Thermal efficiency of a heat engine $\quad \eta_{Carnot} = \left(1 - \dfrac{Q_L}{Q_H}\right)_{Carnot} = 1 - \dfrac{T_L}{T_H}$
- COP of a refrigerator $\quad \beta = \left(\dfrac{Q_L}{Q_H - Q_L}\right)_{Carnot} = \dfrac{T_L}{T_H - T_L}$
- COP of a heat pump $\quad \beta' = \left(\dfrac{Q_H}{Q_H - Q_L}\right)_{Carnot} = \dfrac{T_H}{T_H - T_L}$

The Efficiency of a Carnot Cycle

- Proposition I – it is impossible to construct an engine that operates between two given reservoirs and is more efficient than a reversible engine operating between the same two reservoirs: $\boldsymbol{\eta_{any} \leq \eta_{rev}}$

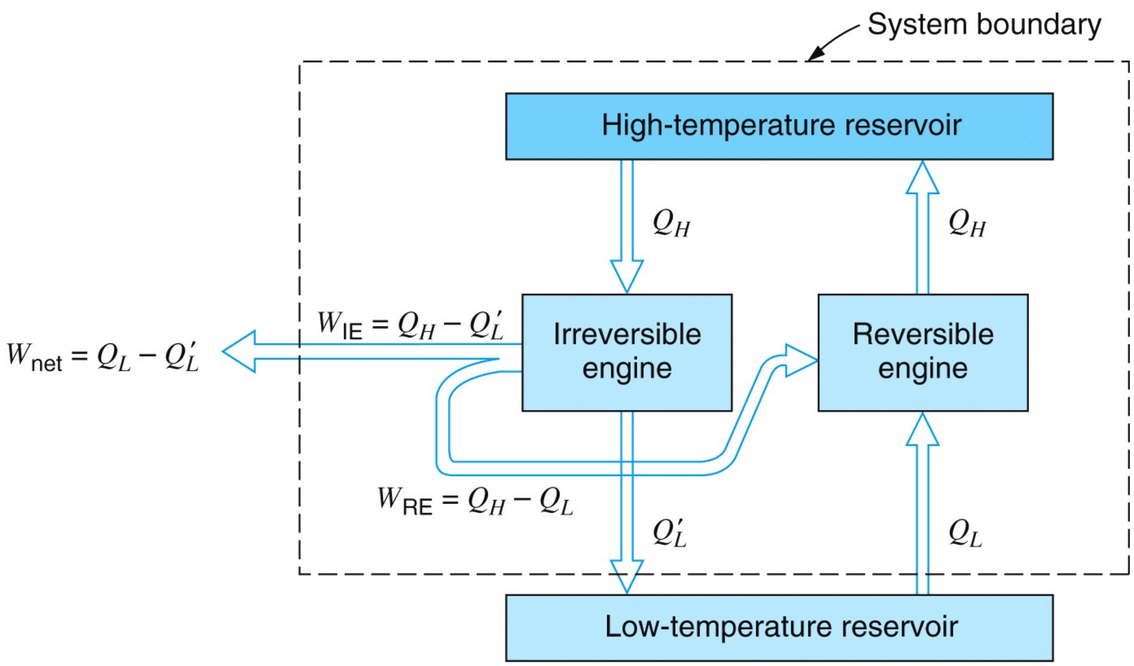


Figure 5.20

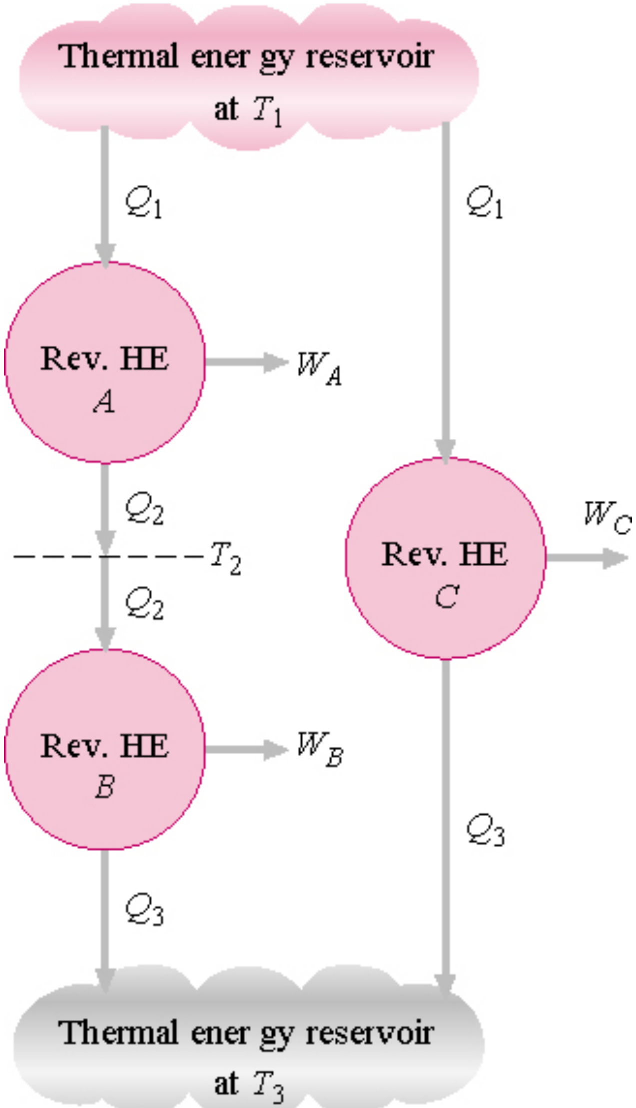
$$\eta_{real} \leq \eta_{Carnot}$$
$$\beta_{real} \leq \beta_{Carnot}$$
$$\beta'_{real} \leq \beta'_{Carnot}$$

- Proposition II – All engines that operate on the Carnot cycle between two given constant-temperature reservoirs have the same efficiency:
  $\boldsymbol{\eta_{rev1} = \eta_{rev2}}$

✓ These propositions suggest that the efficiency of the Carnot cycle operating between two given constant-temperature reservoirs is same regardless of the detail of the system (e.g. working fluid) and it is *a function only of the temperatures.*

$$\eta_{Carnot} = 1 - \frac{Q_L}{Q_H} = 1 - \psi(T_L, T_H)$$

The Thermodynamic Temperature Scale



$$\eta_{thermal} = 1 - \frac{Q_L}{Q_H} = 1 - \psi(T_L, T_H)$$

$$\frac{Q_1}{Q_2} = \psi(T_1, T_2), \quad \frac{Q_2}{Q_3} = \psi(T_2, T_3), \quad \frac{Q_1}{Q_3} = \psi(T_1, T_3)$$

$$\frac{Q_1}{Q_3} = \frac{Q_1 Q_2}{Q_2 Q_3} \rightarrow \psi(T_1, T_3) = \psi(T_1, T_2) \times \psi(T_2, T_3)$$

$$\psi(T_1, T_3) = \psi(T_1, T_2) \times \psi(T_2, T_3) = \frac{f(T_1)}{f(T_2)} \frac{f(T_2)}{f(T_3)}$$

$$\text{,where } \psi(T_1, T_2) = \frac{f(T_1)}{f(T_2)}, \quad \psi(T_2, T_3) = \frac{f(T_2)}{f(T_3)}$$

$$\frac{Q_1}{Q_3} = \psi(T_1, T_3) = \frac{f(T_1)}{f(T_3)} \rightarrow \frac{Q_H}{Q_L} = \frac{f(T_H)}{f(T_L)}$$

Thermodynamic temperature: $\mathbf{T} = Cf(T)$

$$\frac{Q_H}{Q_L} = \frac{\mathbf{T}_H}{\mathbf{T}_L} \qquad \eta_{thermal} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{\mathbf{T}_L}{\mathbf{T}_H}$$

$$\mathbf{T} = \mathbf{T}_0 \frac{Q}{Q_0}$$

In page 5,

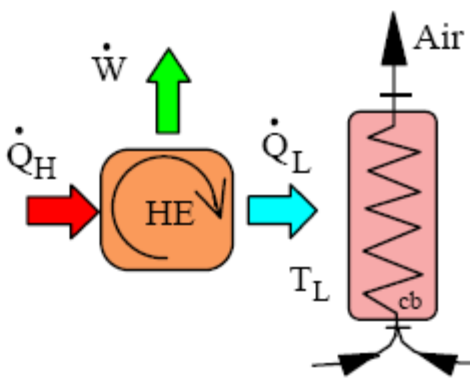
$$\frac{\boldsymbol{Q_H}}{\boldsymbol{Q_L}} = \frac{RT_H \ln \frac{v_2}{v_1}}{RT_L \ln \frac{v_3}{v_4}} = \frac{T_H}{T_L}$$

(ideal gas)

- ✓ Therefore, thermodynamic temperature scale is identical to the ideal-gas temperature scale.

Example 1

A power plant generates 150 MW of electrical power. It uses a supply of 1000 MW from a geothermal source and rejects energy to the atmosphere. Find the power to the air and how much air should be flowed to the cooling tower (kg/s) if its temperature cannot be increased more than 10°C.



C.V. Total power plant.

Energy equation gives the amount of heat rejection to the atmosphere as

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1000 - 150 = \mathbf{850\ MW}$$

The energy equation for the air flow that absorbs the energy is

$$\dot{Q}_L = \dot{m}_{air}\, \Delta h = \dot{m}_{air}\, C_p\, \Delta T$$

$$\dot{m}_{air} = \frac{\dot{Q}_L}{C_p \Delta T} = \frac{850 \times 1000}{1.004 \times 10} = \mathbf{84\ 661\ kg/s}$$

Probably too large to make, so some cooling by liquid water or evaporative cooling should be used.

Example 2

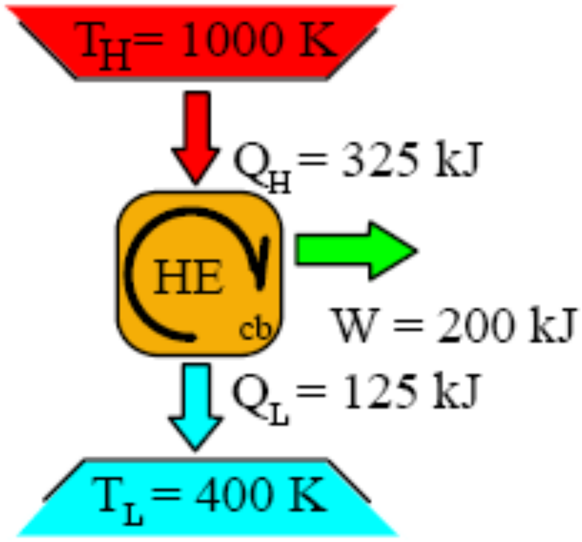
A cyclic machine, shown in Fig. P7.50, receives 325 kJ from a 1000 K energy reservoir. It rejects 125 kJ to a 400 K energy reservoir and the cycle produces 200 kJ of work as output. Is this cycle reversible, irreversible, or impossible?

Solution:

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1000} = 0.6$$

$$\eta_{eng} = \frac{W}{Q_H} = \frac{200}{325} = 0.615 > \eta_{Carnot}$$

This is **impossible.**

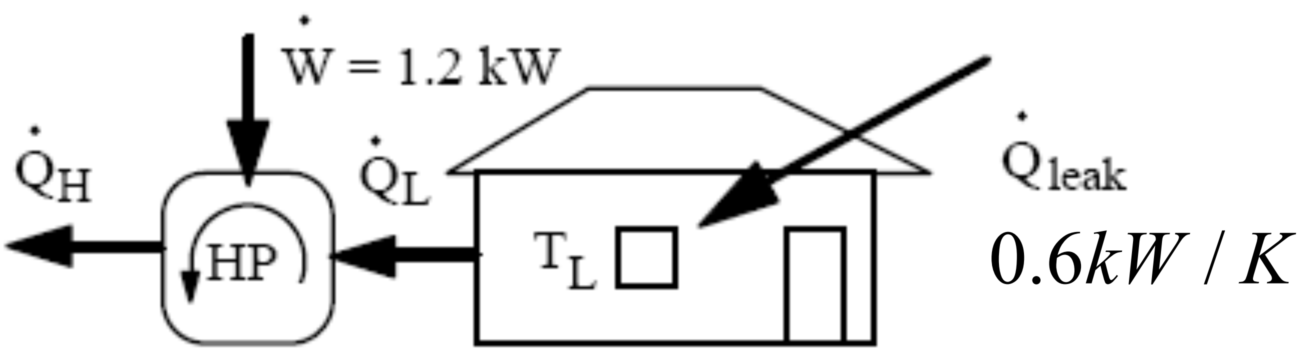


Example 3

An air conditioner cools a house at $T_L = 20°C$ with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the refrigeration COP is $\beta = 0.6\ \beta_{Carnot}$. Find the maximum outside temperature, T_H, for which the air conditioner provides sufficient cooling.

$T_L = T_{house}$

$T_H = T_{amb}$



In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$$\boxed{\dot{Q}_{leak} = 0.6\,(T_{amb} - T_{house}) = \dot{Q}_L}$$ which must be removed by the heat pump.

$$\boxed{\beta = \dot{Q}_L / \dot{W} = 0.6\,\beta_{carnot} = 0.6\,T_{house} / (T_{amb} - T_{house})}$$

$$Q_L / (Q_H - Q_L)$$
$$= T_L / (T_H - T_L)$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})\dot{W}$:

$$0.6\,(T_{amb} - T_{house})^2 = 0.6\,T_{house}\,\dot{W}$$

Since $T_{house} = 293.15$ K and $\dot{W} = 1.2$ kW it follows

$$(T_{amb} - T_{house})^2 = T_{house}\,\dot{W} = 293.15 \times 1.2 = 351.78\ \text{K}^2$$

Solving $\Rightarrow$ $(T_{amb} - T_{house}) = 18.76$ $\Rightarrow$ $T_{amb} =$ **311.9 K = 38.8 °C**